Dsp Ue

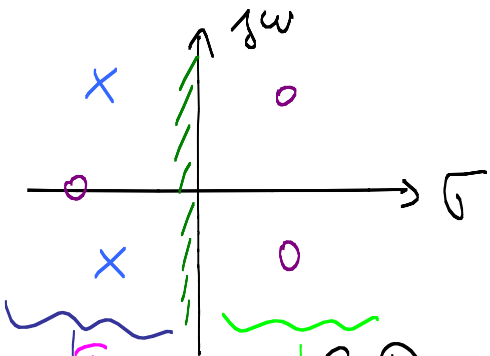
Note Title 23.06.2008

Problem Class 8:

bilinearisation: map a analog into a digital system and vice verse.

Problem Class 8.1.)

analogue system:



$s = \sigma + j\omega$

stable ⟺ all zeros left

bi-linear Transform

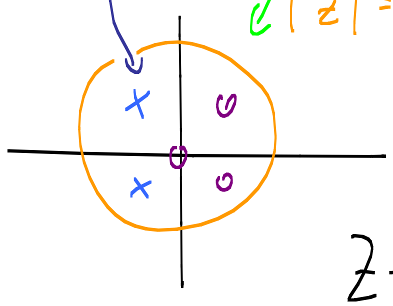
S-Domain (Laplace transformation)

mapped into unity circle

digital System:

mapped outside

$|z| = 1$



$z = \rho e^{j\Theta}$

Z-Domain

bilinear-transform

$$S = \frac{2}{Td} \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$$

$$\omega = \frac{2}{Td} \cdot \tan\left(\frac{\Theta}{2}\right)$$

Td ... sampling periode

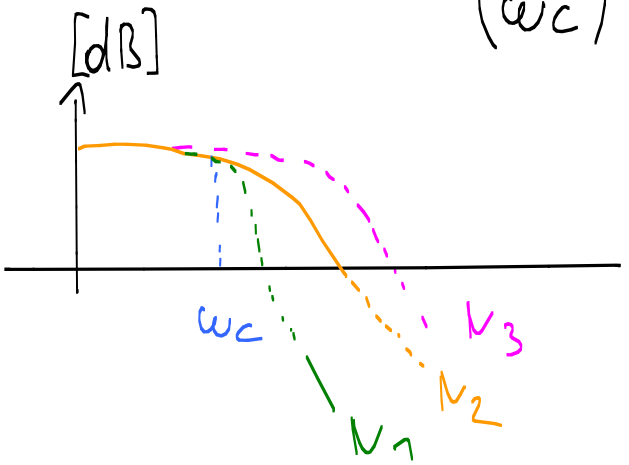
$+\pi \Rightarrow +\infty$

$-\pi \Rightarrow -\infty$

Butterworth Filter:

$$|H_c(j\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_c}\right)^{2N}}$$

N... order of filter



$N_1 > N_2 > N_3$

↳ more poles & zeros = more complex
⇒ higher Order

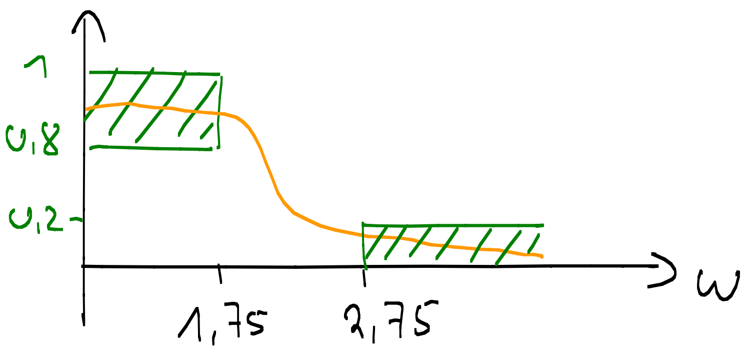
$$H_c(s) = \frac{\omega_c^{\,N}}{\prod_{k=0}^{N-1}(s - s_{p,k})}$$

↳ Pole
⟶ Index

$$s_{p,k} = \omega_c\, e^{j\frac{\pi}{2}\left(1+\frac{1+2k}{N}\right)} \quad 0 \le k \le N$$

Θ	ω
0	0
$0{,}4\pi$	1,45
$0{,}6\pi$	2,75
π	∞

$\omega = \frac{2}{T_d}\tan\left(\frac{\Theta}{2}\right)$ $T_d = 1$ ... for normalized frequencys



to the butterworth filter:

$$1+\left(\frac{\omega_1}{\omega_c}\right)^{2N} = \frac{1}{\left(\left|H_c(j\omega)\right|\,\Big|_{\omega=\omega_1}\right)^2}$$

$$1+\left(\frac{\omega_2}{\omega_c}\right)^{2N} = \frac{1}{\left(\left|H_c(j\omega)\right|\,\Big|_{\omega=\omega_2}\right)^2}$$

in particular:

$$\text{I.)}\quad 1+\left(\frac{1{,}45}{\omega_c}\right)^{2N} = \frac{1}{0{,}8^2}$$

$$\text{II.)}\quad 1+\left(\frac{2{,}75}{\omega_c}\right)^{2N} = \frac{1}{0{,}2^2}$$

$$\Rightarrow N = 2{,}94 \text{ need to be } \in \mathbb{N} \Rightarrow N = 3$$

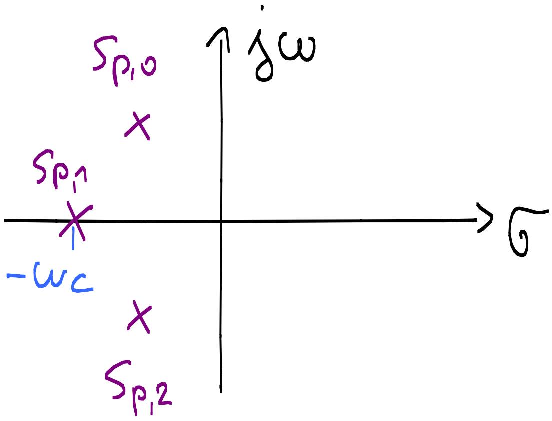
$$\text{II.)}\quad 1+\left(\frac{2{,}75}{\omega_c}\right)^{6} = \frac{1}{0{,}2^2} \Rightarrow \omega_c = 1{,}62$$

Choose I or II because N can't be choosen exactly

$$k=0:\quad s_{p,0} = 1{,}62\cdot e^{j\frac{\pi}{2}\left(1+\frac{1}{3}\right)} = \omega_c\, e^{j\frac{2}{3}\pi}$$

$$k=1:\quad s_{p,1} = 1{,}62\cdot e^{j\frac{\pi}{2}(1+1)} = \omega_c\, e^{j\pi} = -\omega_c$$

$$k=2:\quad s_{p,2} = 1{,}62\, e^{j\frac{\pi}{2}\left(1+\frac{5}{3}\right)} = \omega_c\, e^{j\frac{4}{3}\pi} = \omega_c\, e^{-j\frac{2}{3}\pi}$$



$$H_c(s) = \frac{1{,}63^3}{\left(s - 1{,}62\, e^{j\frac{2}{3}\pi}\right)\left(s + 1{,}62\right)\left(s - 1{,}62\, e^{-j\frac{2}{3}\pi}\right)}$$

$$s = 2 \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$$

$$H(z) = \frac{1{,}62^3}{\left(2\frac{1-z^{-1}}{1+z^{-1}} - 1{,}62\, e^{j\frac{2}{3}\pi}\right)\left(2\frac{1-z^{-1}}{1+z^{-1}} + 1{,}62\right)\left(2\frac{1-z^{-1}}{1+z^{-1}} - 1{,}62\, e^{-j\frac{2}{3}\pi}\right)}$$

$$= \frac{4{,}26\left(1 + z^{-1}\right)^3}{\left(3{,}39\, z^{-2} - 2{,}745\, z^{-1} + 9{,}87\right)\left(-0{,}37\, z^{-1} + 3{,}62\right)}$$

Poles: $z_{p,0} = 0{,}105$

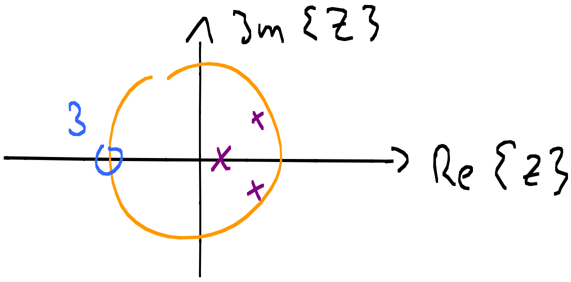
$z_{p,1} = 0{,}13 + j\,0{,}57$

$z_{p,2} = 0{,}13 - j\,0{,}57$

Zeros: $z_{N,0} = 1$

$z_{N,1} = 1$

$z_{N,2} = 1$



Using an analog prototype (here Butterworth)
with a known transferfunction;
transfer into digital ⇒ fit requirements.
in Reality: need to know Td !

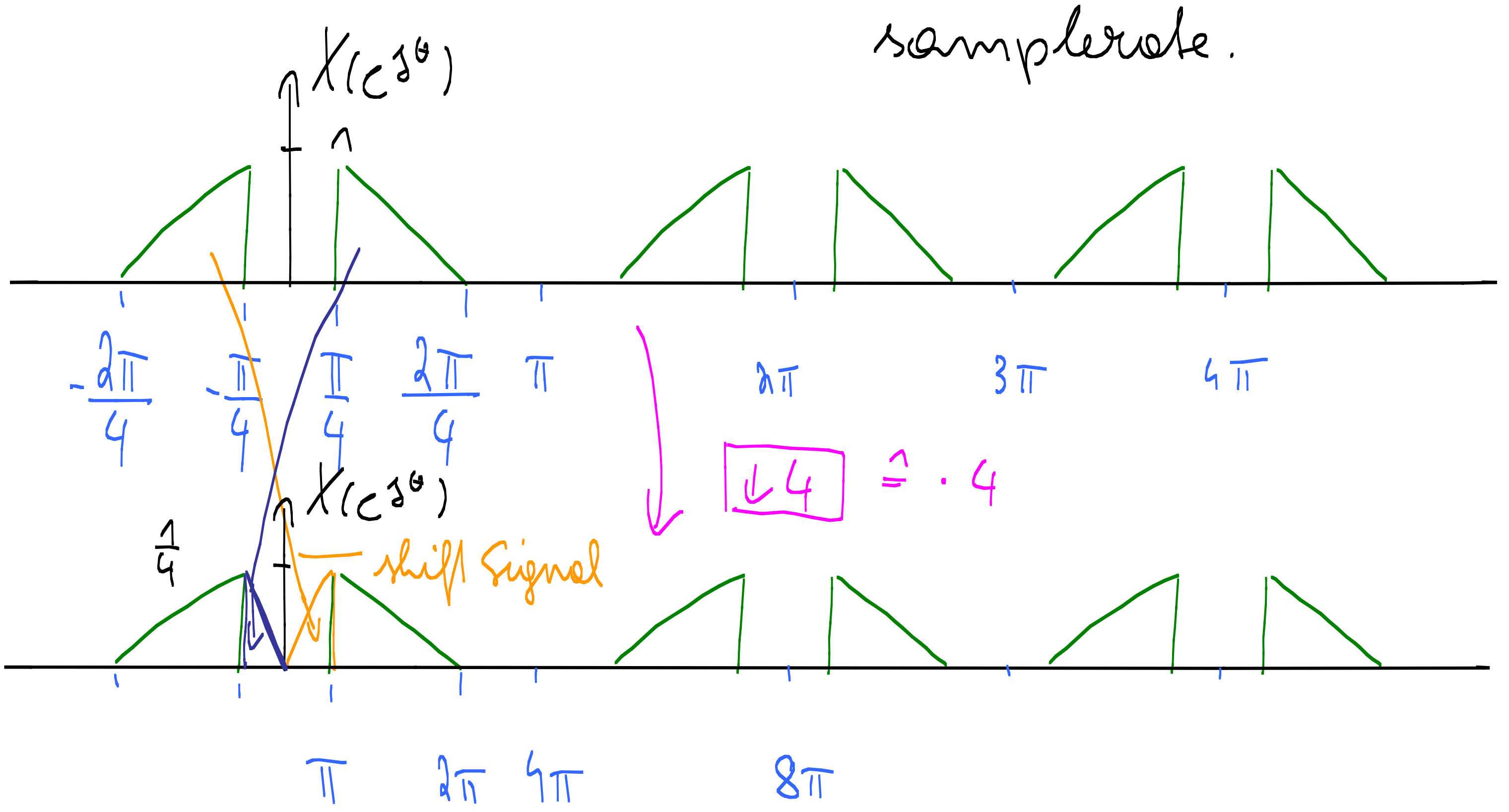
Problem class 8.2:

$x[n] \longrightarrow \boxed{\downarrow 4}$ ... downsampling
here just keep every 4^{th} sample

$\boxed{\uparrow 4}$ ... upsampeling
here just insert 3 zeros between
2 samples

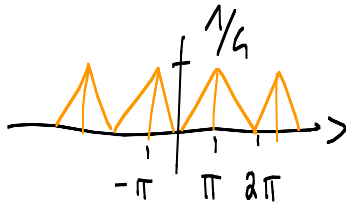
$x[n] \longrightarrow \boxed{\downarrow 4} \longrightarrow \boxed{\uparrow 4} \longrightarrow x_2[n]$

$x[n]$ and $x_2[n]$ will have the same samplerate.



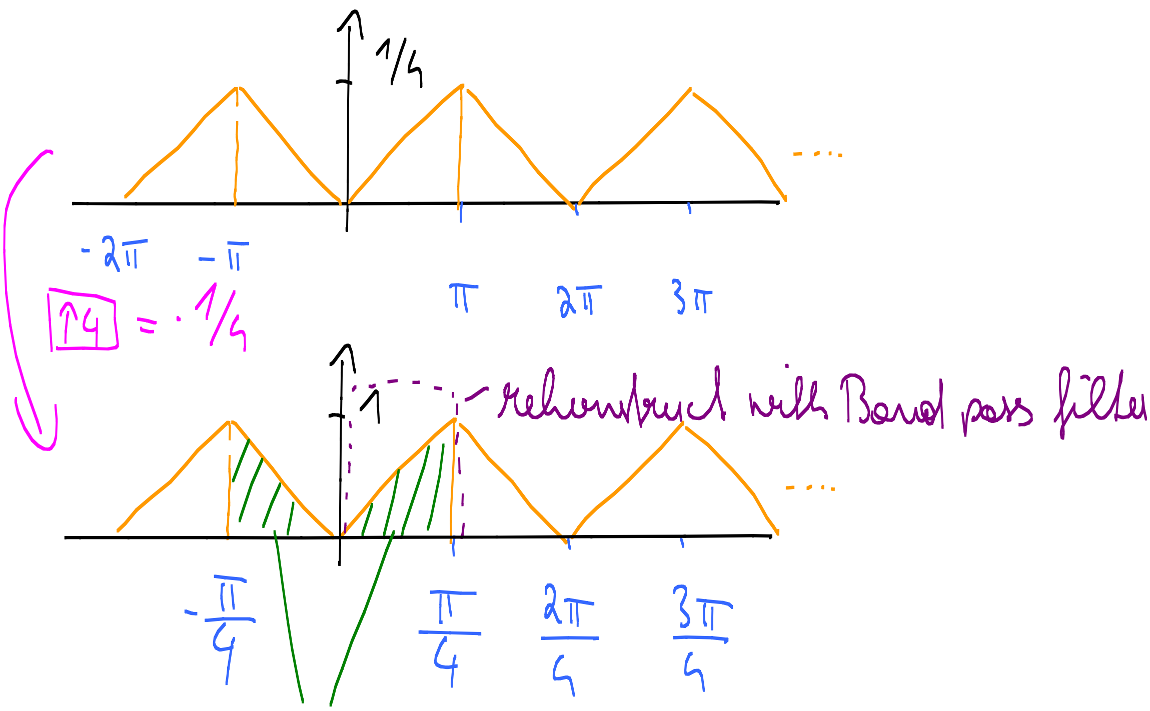
downsample: • lose of energy ⇒ lose of Amplitude
• Spektrum will stay the same

⇒ shift to the left & to the right

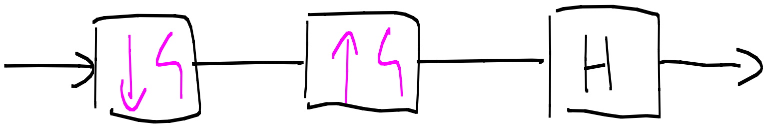


Signal will look total different in time-Domain, but structure will stay the same

Rekonstruktion:



up sampeling adds alliasing to the signal



$x[n] = \cos\left(\frac{\pi}{8} n\right)$ ... repeat after 16 samples

$h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$