NT - Uebung

Note Title 29.04.2008

Phasenmodulation allgemein:

$$u(t) = U(t) \cdot \cos[\Omega(t) \cdot t + \varphi(t)]$$

Amplituden-Modulation, Frequenz-Modulation, Phasen-Modulation

Bspl.) $u(t) = U_0 \cdot \cos[\Omega_0 t + \eta \sin(\omega_1 t + \varphi_1)]$

Formeln nur für cos def → Korrektur

$\varphi_1 = \varphi_1 - 90° = -30°$

$$u(t) = \sum_{n=-\infty}^{\infty} U_0 \cdot I_n(\eta) \cdot \cos[(\Omega_0 + \eta\omega_1)t + n\varphi_1]$$

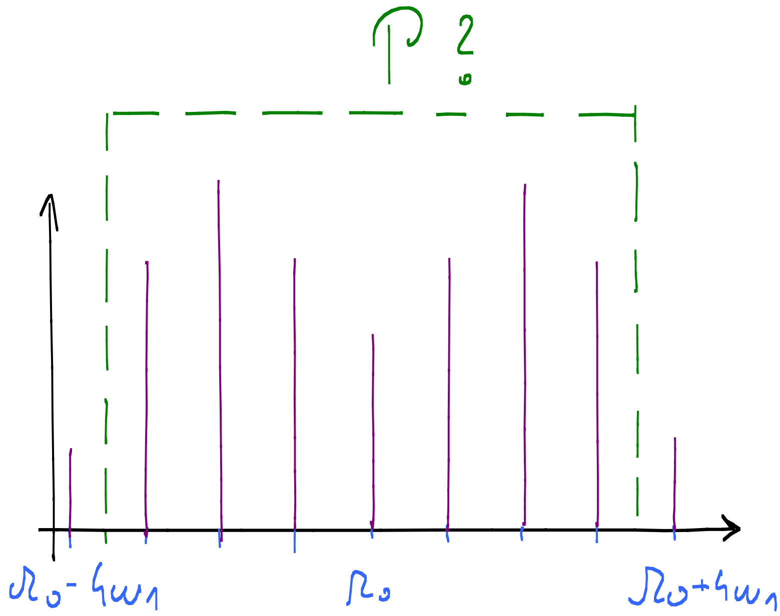
$$= \sum_{n=-\infty}^{\infty} 10 \cdot I_n(3) \cdot \cos[(\Omega_0 + 3\omega_1)t + n(-30)]$$

aus Besseltabelle kommen nur pos Werte

⇒ Bessel für neg. Werte:

$$I_{-n}(\eta) = (-1)^n \cdot I_n(\eta)$$

n	$I_{n(3)}$	$\lvert U \rvert$ \ V
-4	0,132	
-3	-0,3091	
-2	-0,4861	
-1	-0,3391	
0	-0,2601	
1	0,3391	
2	0,4861	
3	0,3091	
4	0,1320	



$$P = U_0^2 \cdot \sum_{n=-\infty}^{\infty} 10 \cdot I_{n(3)}^2 = U_0^2 = P_{ges} \text{ an } 1\Omega$$

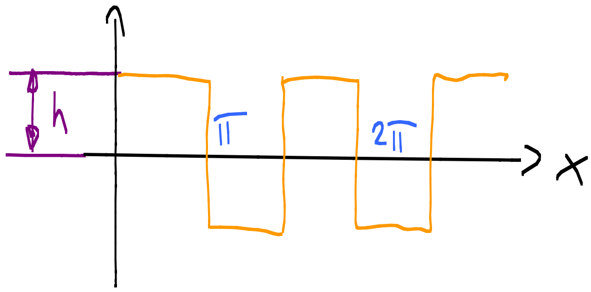
$$P_{\pm 3\omega_1} = U_0^2 \cdot \sum_{n=-\infty}^{\infty} I_{n(3)}$$

$$= 10 \cdot [I_0^2{}_{(3)} + 2 I_1^2{}_{(3)} + 2 I_2^2{}_{(3)} + 2 I_3^2{}_{(3)}]$$

↳ pos und neg bei ² ident

$$\left.\begin{aligned} &= 96{,}13\,W \\ P_{ges} &: U_0^2 = 100\,W \end{aligned}\right\} \quad \frac{P_{\pm 3\omega_1}}{P_{ges}} = 96{,}13\,\%$$

Beispiel 2:



$T = 2\,ms \Rightarrow f_1 = 500\,Hz$

$$f(x) = \frac{4h}{\pi}\left(\sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \dots\right)$$

$$h = U_1 \,;\; \frac{T}{t} = \frac{2\pi}{x} \Rightarrow x = \frac{2\pi}{T}\cdot t = 2\pi f_1 t$$

$$s(t) = \frac{4U_1}{\pi}\left(\sin(2\pi f_1 t) + \frac{1}{3}\sin(3\, 2\pi f_1 t) + \frac{1}{5}(\sin(5\cdot 2\pi f_1 t)) + \dots\right)$$

$\longrightarrow$ Kommt nicht durch Filter

Filter: $f_g = 2\,kHz$: $f_1 = 500\,Hz$

$3f_1 = 1500\,Hz$

$5f_1 = 2500\,Hz > f_g$

$$s(t) = \frac{4U_1}{\pi}\left(\sin(2\pi f_1 t)\right) + \frac{1}{3}\sin(6\pi f_1 t))$$

allgemeine Formeln:

Prinzip

$\Phi(t) = \Omega_0 t + a\cdot s(t) \longrightarrow |s(t)|_{max}$

$\Delta\varphi_{PM} = \eta = |a\cdot s(t)|_{max} \longrightarrow \left|\frac{ds(t)}{dt}\right|_{max}$

$\Delta\Omega_{PM} = \left|a\cdot\frac{ds(t)}{dt}\right|$

Bestimmen von $|s(t)|_{max}$

$$\frac{ds(t)}{dt} = \frac{4U_1}{\pi}\left(\omega_1 \cos(\omega_1 t) + \frac{1}{\cancel{3}}(\cancel{3}\omega_1 \cos(3\omega_1 t))\right)$$

$$= \frac{4U_1\omega_1}{\pi}\left(\cos(\omega_1 t) + \cos(3\omega_1 t)\right) \stackrel{!}{=} 0$$

$$\Rightarrow \cos(\omega_1 t) = -\cos(3\omega_1 t)$$

$$\cos(3\alpha) = 4\cos^3(\alpha) - 3\cos(\alpha)$$

$$\cos(\omega_1 t) = 3\cos(\omega_1 t) - 4\cos^3(\omega_1 t)$$

$$2\cos(\omega_1 t) = 4\cos^3(\omega_1 t) \quad | : 2\cos(\omega_1 t)$$

$$1 = 2\cos^2(\omega_1 t)$$

$$\frac{1}{2} = \cos^2(\omega_1 t)$$

$$\omega_1 t = \arccos\left(\pm\frac{1}{\sqrt{2}}\right) \begin{cases} \pm 135° \\ \pm 45° \end{cases}$$

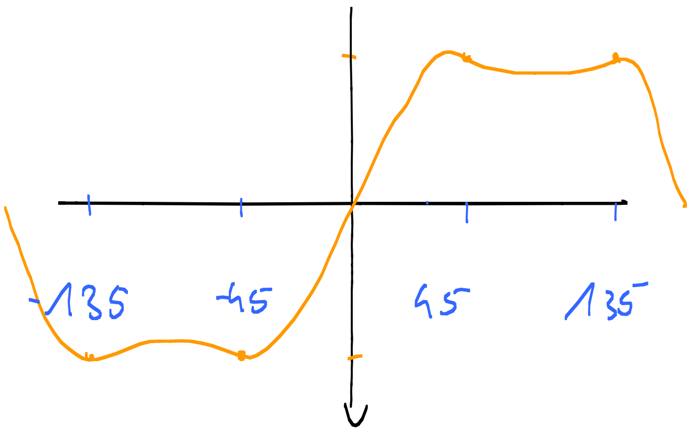
$$\frac{d^2 s(t)}{dt^2} = \frac{4U_1\omega_1}{\pi}\left(-\omega_1 \sin(\omega_1 t) - 3\omega_1 \sin(3\omega_1 t)\right)$$

$$= \frac{4U_1\omega_1^2}{\pi}\left(-\sin(\omega_1 t) - 3\sin(3\omega_1 t)\right)$$

$\omega_1 t$	$\frac{d^2 s(t)}{dt^2}$		
$+45$	$-2,83$	< 0	$\Rightarrow$ max
-45		> 0	$\Rightarrow$ min
$+135$		< 0	$\Rightarrow$ max
-135		> 0	$\Rightarrow$ min

$$|s(t)|_{max} = \frac{U_1 4}{\pi}\left(\underbrace{\sin(45)}_{\frac{1}{\sqrt{2}}} + \frac{1}{3}\underbrace{\sin(135)}_{\frac{1}{\sqrt{2}}}\right)$$

$$= \frac{16 U_1}{3\pi\sqrt{2}} = 1,2004\, U_1$$



Berechnen nun $\left|\frac{ds(t)}{dt}\right|_{max}$

$$\frac{ds(t)}{dt} = \frac{4U_1}{\pi}\left[\underbrace{\cos(\omega_1 t)}_{1} + \underbrace{\cos(3\omega_1 t)}_{1}\right]$$

bei $\omega_1 t = 0 \Rightarrow$ cos ist maximal

$$\left|\frac{ds(t)}{dt}\right|_m = \frac{8U_1\omega_1}{\pi} = \frac{8U_1 2\cancel{\pi} f_1}{\cancel{\pi}} = 16U_1f_1$$

$$a = \frac{\Delta\Omega_{PM}}{\left|\frac{ds(t)}{dt}\right|_{max}}$$

$$\Delta\Phi = |a \cdot s(t)|_{max} = \frac{\Delta\Omega_{PM}}{\left|\frac{ds(t)}{dt}\right|_{max}} \cdot |s(t)|_{max}$$

$$\frac{2\cancel{\pi}\ 10\,kHz}{\cancel{16}\ \cancel{U_1} \cdot 0{,}5\,kHz} \cdot \frac{\cancel{16}\cancel{U_1}}{3\cancel{\pi}\sqrt{2}} = 9{,}428 \approx 3\pi$$

$$\hat{=}\ 540^\circ$$