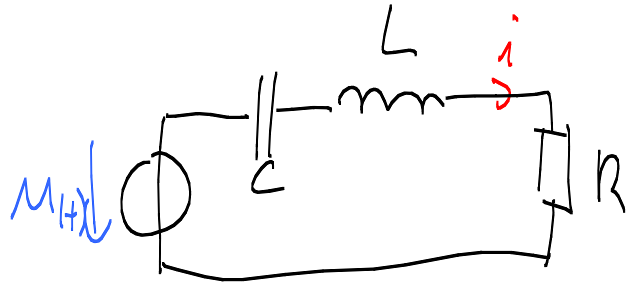
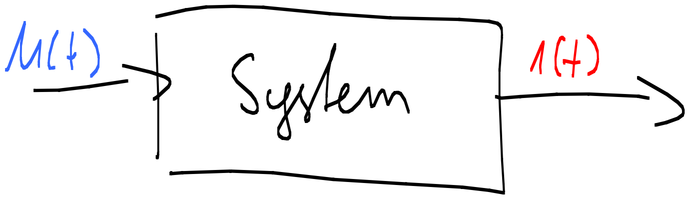
Nes

Note Title 31.03.2008

Test 08.06.2007, Doudomas

1a.)




$m_1$: $0 = u_C + u_L + u_R - u(t)$

$$0 = \frac{1}{C}\int i(t)\,dt + L\frac{di}{dt} + i\,R - u(t) \quad \Big/ \frac{d}{dt}$$

$$0 = \frac{1}{C}\, i(t) + L\, i''(t) + i'\, R - 0$$

$$0 = i''(t) + \frac{R}{L}\, i'(t) + \frac{1}{LC}\, i(t)$$

$=$

$$u(t) = u_C + L\frac{di}{dt} + i\cdot R \qquad \Big/ \ \vec{x} = \begin{pmatrix} u_C \\ i(t) \end{pmatrix}$$

$$\frac{dx_2}{dt} = \frac{1}{L}\,u(t) - \frac{1}{L}\,x_1 - x_2\cdot\frac{R}{L}$$

aus Bauteil: $i(t) \hat{=} i_C = C \frac{du_C}{dt} = C \frac{dx_1}{dt} = x_2$

$$\Rightarrow \frac{dx_1}{dt} = \frac{1}{C} x_2 + 0 x_1 +$$

$$\Rightarrow A \dot{x} = b \cdot u$$

$$\begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \frac{dx}{dt} = -\frac{1}{L} u(t)$$

$$\underline{y} = c^T x + du \hat{=} i(t) \Rightarrow c^T \begin{pmatrix} u_C \\ i(t) \end{pmatrix} \hat{=} i(t) \Rightarrow c^T = (0 \ 1) + 0u$$

1b.) $\frac{dx}{dt} = A \cdot x \qquad | x(t) = p_0 \cdot e^{sT}$

$$p \cdot s \cdot \cancel{e^{sT}} = A \cdot p \, \cancel{e^{sT}}$$

$$p s = A \cdot p \Rightarrow p \cdot s - A \cdot p = 0$$

$$p (s - A) = 0$$

$$x_1(t) = p_1 e^{s_1 t}$$
$$x_2(t) = p_2 e^{s_2 t} \qquad x(t) = \gamma_1 x_1 + \gamma_1 x_2 \ .. \text{ Linear-kombi.}$$

$$\Rightarrow \gamma_1 p_1 e^{s_1 t} + \gamma_2 p_2 e^{s_2 t}$$

$$[p_1 \ p_2] \begin{bmatrix} e^{s_1 t} & 0 \\ 0 & e^{s_1 t} \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} \quad / \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} = [p_1 \ p_2] x_0$$

$$x(t) = \Phi(t) \cdot x_0$$

$$\Phi(t) = [p_1 \; p_2] \begin{bmatrix} e^{s_1 t} & 0 \\ 0 & e^{s_1 t} \end{bmatrix} [p_1 \; p_2]^{-1}$$

$$\Delta s = |(S - A)| = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 6 \\ -1 & -5 \end{bmatrix} = \begin{bmatrix} S & -6 \\ 1 & S+5 \end{bmatrix}$$

$$\Delta s = \det = (S)(S+5) - (-6)$$

$$= S^2 + 5S + 6$$

$$s = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 (6)}}{2 \cdot 1}$$

$$s_1 = -3 \qquad S_1 \neq S_2$$
$$s_2 = -2$$

$\Rightarrow \exists\, 2$ l.u. Eigenvektoren

$$(S_1 - A)\, p_1 = 0 \Rightarrow \begin{vmatrix} -3 & -6 \\ 1 & -3+5 \end{vmatrix} \begin{pmatrix} p_{11} \\ p_{21} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-3 p_{11} - 6 p_{21} = 0 \Rightarrow p_{11} = 2 p_{21}$$
$$p_{11} + 2 p_{21} = 0$$

$\Rightarrow \infty$ viele Lösungen

$$-3(2 p_{21}) - 6 p_{21} = 0$$

$\hookrightarrow$ wähle $p_{12} = 1$

$p_{11} = -2$

$$p_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\begin{vmatrix} -2 & -6 \\ 1 & -2+5 \end{vmatrix} \begin{pmatrix} p_{21} \\ p_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-2 p_{21} - 6 p_{22} = 0 \Rightarrow p_{21} = -3 p_{22}$$
$$p_{21} + 3 p_{22} = 0 \Rightarrow -3 p_{22} = 3 p_{22};$$

$$\Rightarrow p_2 = \begin{bmatrix} 1 \\ -1/3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 & 1 \\ 1 & -1/3 \end{bmatrix} \cdot \begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 1 & -1/3 \end{bmatrix}^{-1}$$

$$\begin{array}{cc|cc} & & e^{-3t} & 0 \\ & & 0 & e^{-2t} \\ \hline -2 & 1 & -2e^{-3t} & e^{-2t} \\ 1 & -1/3 & e^{-3t} & -1/3\, e^{-2t} \end{array}$$

$$\frac{1}{\det} \begin{vmatrix} -1/3 & -1 \\ -1 & -2 \end{vmatrix}$$

$$\hookrightarrow \frac{2}{3} - 1 = -\frac{1}{3}$$

$$\begin{vmatrix} 1 & 3 \\ 3 & 6 \end{vmatrix}$$

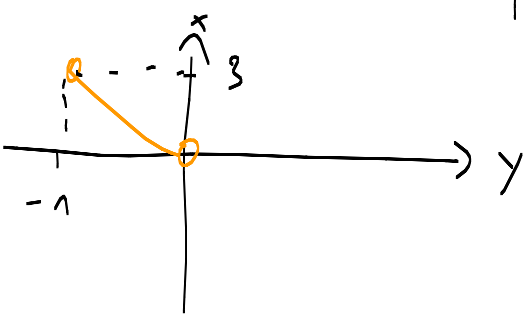
$$\begin{array}{cc|cc} & & 1 & 3 \\ & & 3 & 6 \\ \hline -2e^{-3t} & e^{-2t} & -2e^{-3t} + 3e^{-2t} & -6e^{-3t} + 6e^{-2t} \\ e^{-3t} & -1/3\, e^{-2t} & e^{-3t} - e^{-2t} & 3e^{-3t} - 2e^{-2t} \end{array}$$

1c.)

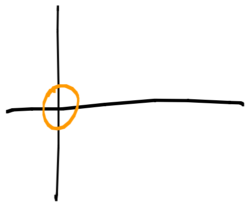
$$x_0^{(1)} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \Rightarrow \quad x(t) = \Phi(t) \cdot x_0^{(1)}$$

$$\begin{array}{cc|c} & & 3 \\ & & -1 \\ \hline (-2e^{-3t} + 3e^{-2t}) & (-6e^{-3t} + 6e^{-2t}) & (-\cancel{6e^{-3t}} + \cancel{9}e^{-2t}) + (\cancel{6e^{-3t}} - \cancel{6e^{-2t}}) \\ (e^{-3t} - e^{-2t}) & (3e^{-3t} - 2e^{-2t}) & (\cancel{3e^{-3t}} - \cancel{3e^{-2t}}) + (\cancel{-3e^{-3t}} + 2\cancel{e}^{-2t}) \end{array}$$

$$\lim_{t \to \infty} \begin{pmatrix} 3e^{-2t} \\ -e^{-2t} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$



$$x_{(0)}^{(2)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \lim_{t \to \infty} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

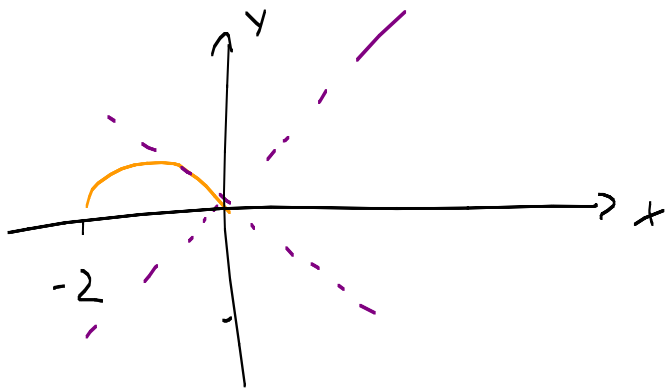


$$x_0^{(3)} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc|c} & & -2 \\ & & 0 \\ \hline (-2e^{-3t}+3e^{-2t}) & (-6e^{-3t}+6e^{-2t}) & (+4e^{-3t}+12e^{-2t}) \\ (e^{-3t}-e^{-2t}) & (3e^{-3t}-2e^{-2t}) & (-2e^{-3t}+2e^{-2t}) \end{array}$$

$$2\begin{pmatrix} 2e^{-3t}+6e^{-2t} \\ -e^{-3t}+e^{-2t} \end{pmatrix}$$

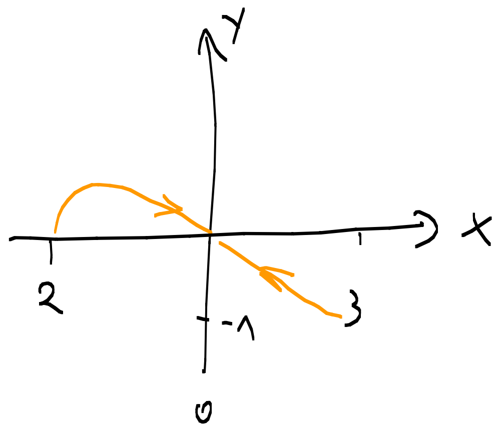
$$\lim_{t \to \infty} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$




$x_{(0)}^{(1)}$

$x_{(0)}^{(2)}$

$x_{(0)}^{(3)}$



2a) $\frac{dx_1}{dt} = x_1^2 + x_1 x_2 - 2 \qquad \frac{dx_2}{dt} = \frac{4}{x_1^2} - 1$

$$\left.\frac{dx_1}{dt}\right|_{RL} = 0 \quad \Rightarrow \quad x_1^2 + x_1 x_2 - 2 \overset{!}{=} 0$$

$$x_1(x_1 + x_2) - 2 \overset{!}{=} 0$$

$$\left.\frac{dx_2}{dt}\right|_{x_{RL}} = 0 \quad \Rightarrow \quad 4 \cdot \frac{1}{x_1^2} - 1 \overset{!}{=} 0 \Rightarrow 4 = x_1^2 \Rightarrow x_1 = \pm 2$$

$$\left.\frac{dx_1}{dt}\right|_{x_{RL}} = 0 \quad \Rightarrow \quad 4 \pm 2x_2 - 2 = 0$$

$$\pm 2x_2 = -2$$

$$x_2 = \pm 1$$

$$x_{RL}^{(1)} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \qquad x_{RL}^{(2)} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

2b.)

$$\frac{d\xi}{dt} = A \cdot \xi$$

$$\left.\frac{\partial f_1}{\partial x_1}\right|_{x_R} = 2x_1 + x_2 \qquad \frac{\partial f_2}{\partial x_1} = 4 \cdot (-2) \frac{1}{x_1^3}$$

$$\left.\frac{\partial f_1}{\partial x_2}\right|_{x_R} = x_1 \qquad \frac{\partial f_2}{\partial x_2} = 0$$

$$\begin{bmatrix} 2x_1 + x_2 & x_1 \\ -8\frac{1}{x_1^3} & 0 \end{bmatrix} \cdot \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}$$

2c.) $(S_E - A) = 0$

$$\begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 2x_1 + x_2 & x_1 \\ -8\frac{1}{x_1^3} & 0 \end{bmatrix} = \begin{bmatrix} S - 2x_1 - x_2 & -x_1 \\ +8\frac{1}{x_1^3} & S \end{bmatrix}$$

$$= \begin{bmatrix} S - 2(2) + 1 & -2 \\ +8\frac{1}{2^3} & S \end{bmatrix}$$

$$= \begin{bmatrix} S - 3 & -2 \\ -1 & S \end{bmatrix}$$

$$(S)(S-3) + (2) = S^2 - 3S + 2$$

$$S_{12} = \frac{+3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (+2)}}{2 \cdot 1} \begin{matrix} \nearrow S_1 = 1 \\ \searrow S_2 = 2 \end{matrix}$$

instabil

$\exists S > 0$

$$\begin{bmatrix} S-2(-2)-1 & +2 \\ 8\ \frac{1}{2^3} & S \end{bmatrix} = \begin{bmatrix} S+3 & +2 \\ -1 & S \end{bmatrix}$$

$$\det |\,| = S(S+3)-2 = S^2+3S-3$$

$$S_{1,2} = \frac{-3 \pm \sqrt{3^2-4\cdot 1\cdot(-2)}}{2\cdot 1}$$

$S_1 = -2$ | asymptotisch

$S_2 = -1$ | grenzstabil

da $\exists S_1, S_2 < 0$

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1a.) $\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\mathbf{x}$

$$(S_E - A) = 0$$

$$\begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} S-1 & 0 \\ 0 & S+1 \end{bmatrix}$$

$$(S-1)(S+1) = S^2-1 = 0$$

$S_1 = 1$

$S_2 = -1$

$$x(t) = (S_E - A)\cdot \vec{r_0} = 0$$

$$(S_{1E} - A)\begin{pmatrix} r_{011} \\ r_{012} \end{pmatrix} = 0$$

$$\left( \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \right) \cdot \begin{pmatrix} p_{11} \\ p_{12} \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} p_{11} \\ p_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$0 \cdot p_{11} + 2 \cdot p_{12} = 0 \quad \Rightarrow p_{12} = 0$

$p_{11}$ frei wählbar $\Rightarrow 1$

$$\vec{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$(S_2 - A) \cdot p_2 = 0$$

$$\left( \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} - \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \right) \begin{pmatrix} p_{21} \\ p_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{vmatrix} -2 & 0 \\ 0 & 0 \end{vmatrix} \cdot \begin{pmatrix} p_{21} \\ p_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$-2 p_{21} + 0 p_{22} = 0 \quad \Rightarrow \quad p_{21} = 0$

$0 p_{21} + 0 p_{22} = 0 \quad \Rightarrow \quad p_{22}$ frei wählbar $= 1$

$$\vec{p_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

1b.) $\Phi(t) = \vec{p} \cdot \operatorname{diag}(e^{S_i t}) \, \vec{p}^{\,-1}$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{1t} & 0 \\ 0 & e^{-1t} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{array}{cc|cc} & & e^{1t} & 0 \\ & & 0 & e^{-1t} \\ \hline 1 & 0 & e^{t} & 0 \\ 0 & 1 & 0 & e^{-t} \end{array}$$

$$\begin{array}{cc|cc} & & 1 & 0 \\ & & 0 & 1 \\ \hline e^{t} & 0 & e^{t} & 0 \\ 0 & e^{-t} & 0 & e^{-t} \end{array}$$

1c.) $\mathbf{x}_0^{(1)} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$,

$$x(t) = \Phi x_0^{(1)} = \begin{array}{cc|c} & & 3 \\ & & 0 \\ \hline e^{t} & 0 & 3e^{t} \\ 0 & e^{-t} & 0 \end{array} \qquad \lim_{t\to\infty} = \begin{bmatrix} \infty \\ 0 \end{bmatrix}$$

$\mathbf{x}_0^{(2)} = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$,

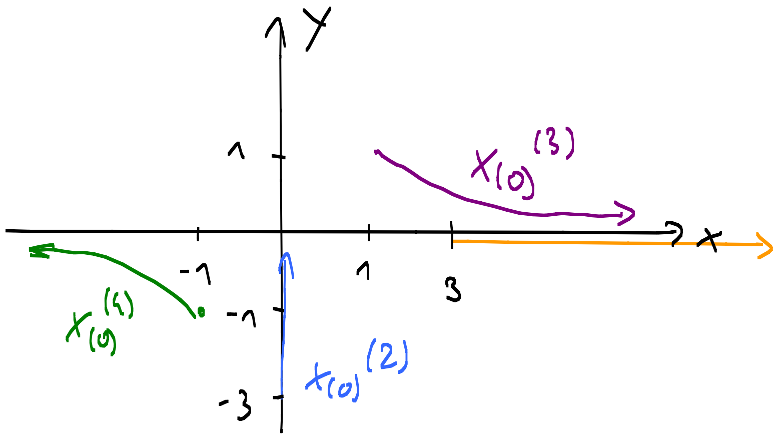
$$x(t) = \Phi x_0^{(2)} = \begin{array}{cc|c} & & 0 \\ & & -3 \\ \hline e^{t} & 0 & 0 \\ 0 & e^{-t} & -3e^{-t} \end{array} \qquad \lim_{t\to\infty} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\mathbf{x}_0^{(3)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$,

$$x(t) = \Phi x_0^{(3)} = \begin{array}{cc|c} & & 1 \\ & & 1 \\ \hline e^{t} & 0 & e^{t} \\ 0 & e^{-t} & e^{-t} \end{array} \qquad \lim_{t\to\infty} = \begin{bmatrix} \infty \\ 0 \end{bmatrix}$$

$\mathbf{x}_0^{(4)} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$$x(t) = \Phi x_0^{(3)} = \begin{array}{cc|c} & & -1 \\ & & -1 \\ \hline e^{t} & 0 & -e^{t} \\ 0 & e^{-t} & -e^{-t} \end{array} \qquad \lim_{t\to\infty} = \begin{bmatrix} -\infty \\ 0 \end{bmatrix}$$




2a.)

$$\frac{dx_1}{dt} = -x_1 + 3x_2^2 + u$$
$$\frac{dx_2}{dt} = x_2^2 + 4x_2 + 3 + u^2$$

$$u_R = 0$$

$$\left.\frac{dx_1}{dt}\right|_{x_R} = 0 \Rightarrow -x_1 + 3x_2^2 = 0$$

$$\left.\frac{dx_2}{dt}\right|_{x_R} = 0 \Rightarrow x_2^2 + 4x_2 + 3 = 0$$

$$x_{2_{1,2}} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$$

$$x_{2,1} = -1$$

$$x_{2,2} = -3$$

$$\left.\frac{dx_1}{dt}\right|_{x_R} = 0 \Rightarrow x_{1,1} = 3(-1)^2 = 3$$
$$x_{1,2} = 3(-3)^2 = 27$$

$$x_{RL}^{(1)} = \begin{array}{c}\begin{pmatrix} 3 \\ -1 \end{pmatrix}\end{array}\begin{array}{l} x_1 \\ x_2 \end{array} \qquad x_{RL}^{(2)} = \begin{array}{c}\begin{pmatrix} 27 \\ -3 \end{pmatrix}\end{array}\begin{array}{l} x_1 \\ x_2 \end{array}$$

2b.) $\frac{dx_1}{dt} = -x_1 + 3x_2^2 + u \rightarrow f_1$

$\frac{dx_2}{dt} = x_2^2 + 4x_2 + 3 + u^2 \rightarrow f_2$

$$\frac{\partial f_1}{\partial x_1} = -1 \qquad \frac{\partial f_1}{\partial x_2} = 3 \cdot 2 \cdot x_2 \qquad \frac{\partial f_1}{\partial u} = 1$$

$$\frac{\partial f_2}{\partial x_1} = 0 \qquad \frac{\partial f_2}{\partial x_2} = 2x_2 + 4 \qquad \frac{\partial f_2}{\partial u} = 2u$$

$$\begin{bmatrix} -1 & 6x_2 \\ 0 & 2x_2+4 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2u \end{bmatrix} \cdot v$$

2c.) $(s_E - A) = 0$

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 6x_2 \\ 0 & 2x_2+4 \end{bmatrix} = \begin{bmatrix} s+1 & -6x_2 \\ 0 & s-2x_2-4 \end{bmatrix}$$

$x_2 = -1$ aus $x_{RL}^{(1)}$

$$= \begin{bmatrix} s+1 & -6(-1) \\ 0 & s-2(-1)-4 \end{bmatrix}$$

$$= \begin{bmatrix} s+1 & +6 \\ 0 & s-2 \end{bmatrix}$$

$\nabla$-Matrix: $s_1 = -1$ | instabil

$s_2 = 2$ | da $\exists s_i > 0$

$x_2 = -3$ aus $x_{RL}^{(1)}$

$$= \begin{bmatrix} s+1 & -6(-3) \\ 0 & s-2(-3)-4 \end{bmatrix} = \begin{bmatrix} s+1 & +18 \\ 0 & s+2 \end{bmatrix}$$

$\nabla$ - Matrix: $s_1 = -1$, $s_2 = -2$ | asymptotisch stabil da $\nexists\, s_i \geq 0$

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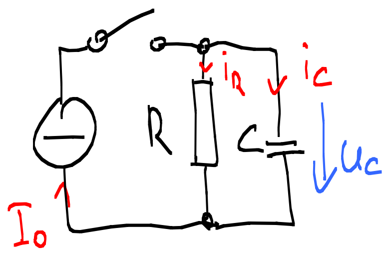
02.02.2008 Teil Magele




$I_0 = 1\text{mA} = \text{const.}$

$R = 1k\Omega$

$C = 100\mu F$

1a.) $I_0 = i_R + i_C$

$$i_R = \frac{u_C}{R}$$

$$i_C = C\frac{du_C}{dt}$$

$$I_0 = \frac{1}{R}u_C + C\frac{du_C}{dt}$$

$$\frac{1}{C}I_0 = \frac{du_C}{dt} + \frac{1}{RC}u_C$$

1b.) Ansatz für homogene Dgl 1. Ordnung

$$u_c = h e^{-\lambda t}$$

$$u_c' = -\lambda h e^{-\lambda t}$$

Homogener Anteil:

$$0 = u_c' + \frac{1}{RC} u_c$$

$$= -\lambda h e^{-\lambda t} + \frac{1}{RC} h e^{-\lambda t}$$

$$= h e^{-\lambda t} \left[ -\lambda + \frac{1}{RC} \right]$$

$\Rightarrow$ Produkt Null: $\lambda = \frac{1}{RC}$

partikulär:

$$u_c = A$$

$$u_c' = 0$$

$$\frac{1}{\cancel{C}} I_0 = 0 + \frac{1}{R\cancel{C}} A$$

$$I_0 \cdot R = A$$

homogen + partikulär:

$$u_{Cp} + u_{Ch} = u_C(t)$$

$$I_0 \cdot R + k e^{-\frac{1}{RC}t} = u_C(t)$$

Startwert $u_C(t=0) = 0V$

$$I_0 \cdot R + k = 0 \Rightarrow k = -I_0 \cdot R$$

Lösung:

$$u_C(t) = I_0 \cdot R + (-I_0 \cdot R) e^{-\frac{1}{RC}t}$$

$$= I_0 \cdot R \left[1 - e^{-\frac{1}{RC}t}\right]$$

1c.) Euler Verfahren:

$$u(t_{k+1}) = u(t_k + h)$$

Taylor: $u(t_k) + \frac{1}{1!} \left.\frac{du}{dt_k}\right|_{t_k} \cdot h$ + Rest vernachlässigbar

Euler: $u(t_{k+1}) \approx u(t_k) + \left.\frac{du}{dt}\right|_{t_k} \cdot h$ ↳ Schrittweite

↳ explizit, forward

Euler: $u(t_{k+1}) \approx u(t_k) + \left.\frac{du}{dt}\right|_{t_{k+1}} \cdot h$

↳ implizit, backward

$$u(t_{k+1}) = u(t_k) + \left.\frac{du_c}{dt}\right|_{t_k} \cdot h$$

$$\left.\frac{du_c}{dt}\right|_{t_k} = \frac{1}{C}\left(I_0 - \frac{1}{R}u_c(t_k)\right)$$

$$u(t_{k+1}) = u(t_k) + \frac{1}{C}\left(I_0 - \frac{1}{R}u_c(t_k)\right) \cdot h$$ ... explizites

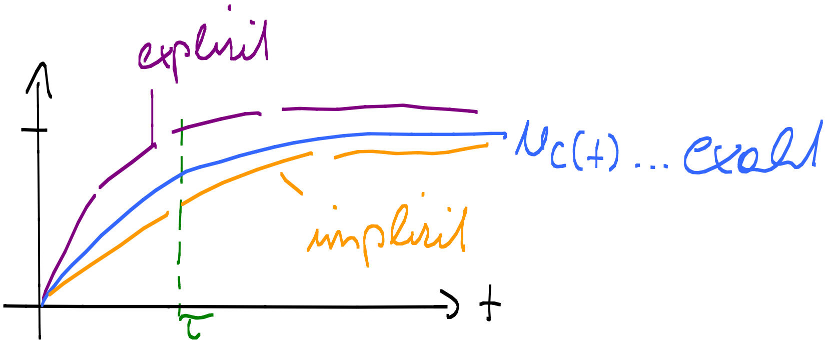
$$\left.\frac{du_c}{dt}\right|_{t_{k+1}} = \frac{1}{C}\left(I_0 - \frac{1}{R}u_c(t_{k+1})\right)$$

$$u(t_{k+1}) = u(t_k) + \frac{1}{C}\left(I_0 - \frac{1}{R}u_c(t_{k+1})\right) \cdot h$$

$$u(t_{k+1}) \cdot \left[1 + \frac{1}{RC}h\right] = u_{t_k} + \frac{1}{C}I_0 h$$

$$u(t_{k+1}) = \left(u_{t_k} + \frac{h}{C}I_0\right)\left(\frac{1}{1 + \frac{1}{RC}h}\right)$$ ... implizit

1d.) $I_0 \cdot R$




explizit liegt immer über dem Wert der exakten Lösung

implizite liegt immer unter dem Wert der exakten Lösung

$\Rightarrow$ Reduktion des Fehlers durch kleinere Schrittweite von h.

1e.) $$u_{C(k+1)}^{*} = u_{C(k)} + h \cdot f(\ldots)$$

$$f(\ldots) = \left.\frac{du_C}{dt}\right|_{t_k} = \frac{1}{C}\left(I_0 - \frac{1}{R} u_{C(t_k)}\right)$$

$$= u_{C(k)} + \frac{h}{C}\left(I_0 - \frac{1}{R} u_{C(k)}\right) \text{ ... Prediktor}$$

$$u_{C(k+1)} = u_{C(k)} + \frac{h}{2C}\left(I_0 - \frac{1}{R} u_{(k)}\right) + \frac{h}{2C}\left(I_0 - \frac{1}{R} u_{C(k+1)^*}\right)$$

$$= u_{C(k)} + \frac{h}{2C}\left[\left(I_0 - \frac{1}{R} u_{(k)}\right) + \left(I_0 \frac{1}{R} u_{C(k+1)^*}\right)\right]$$

... Korrektor

exakte Lösung: $u_{C(t)} = I_0 \cdot R \left[1 - e^{-\frac{1}{RC}t}\right]$

$I_0 = 1\text{mA} = \text{const.}$

$R = 1k\Omega$

$C = 100\mu F$

$\tau = R \cdot C$

$1 \cdot \cancel{10^3} \cdot \cancel{100} \cdot \cancel{10^{-6}}$

$\tau = 0{,}1$

$h = \frac{\tau}{2} = \frac{0{,}1}{2} = 0{,}05$

$t_0 = 0$ : $0$ V

$t_1 = t_0 + h$ : $0{,}3935$ V

$t_2 = t_1 + h$ : $0{,}6321$ V

$u_{(t_k)} + \frac{1}{C}\left(I_0 - \frac{1}{R} u_{C(t_k)}\right) \cdot h$ ... explizit Euler

$t_0 = 0$ : $0{,}5$V

$t_1 = t_0 + h$ : $0{,}75$V

$t_2 = t_1 + h$ : $0{,}875$

$= \left(u_{t_k} + \frac{h}{C} I_0\right)\left(\frac{1}{1 + \frac{1}{RC}h}\right)$ ... implizit Euler

$t_0 = 0$ : $1/3$ V

$t_1 = t_0 + h$ : $0{,}\dot{5}$V

$t_2 = t_1 + h$ : $0{,}\overline{703}$ V

$$u_{c(k+1)}^* = u_{c(k)} + \frac{h}{C}\left(I_0 - \frac{1}{R} u_{c(k)}\right) \quad \text{... Prediktor}$$

$$u_{c(k+1)} = u_{c(k)} + \frac{h}{2C}\left[\left(I_0 - \frac{1}{R} u_{(k)}\right) + \left(I_0 \frac{1}{R} u_{c(k+1)}^*\right)\right] \quad \text{... Korrektor}$$

| | | $u_{c(k+1)}^*$ | $u_{c(k+1)}$ |
|---|---|---|---|
| $t_0 = 0$ | : | 0,5 | 0,375 |
| $t_1 = t_0 + h$ | : | 0,6875 | 0,60999 |
| $t_2 = t_1 + h$ | : | 0,8096 | 0,75586 |

2a.)

$$g(x) = g(x_0) + \frac{1}{1!} \cdot \frac{dg}{dx}\Big|_{x_0} \Delta x \underbrace{+ \frac{1}{2!} \frac{d^2 g}{dx^2}\Big|_{x_0} \cdot \Delta x^2 + \dots}_{\text{Vernachlässigbar}}$$

$$g(x_1) = g(x_0 + \Delta x)$$

$$= g(x_0) + \frac{dg}{dt}\Big|_{x_0} \Delta x \qquad \Big| \ \Delta x = x_1 - x_0$$

$$= g(x_0) + \frac{dg}{dt}\Big|_{x_0} \cdot (x_1 - x_0) \overset{!}{=} 0$$

$$g(x_0) \qquad = \frac{dg}{dt}\Big|_{x_0} (x_1 - x_0)$$

$$x_0 - g(x_0) \frac{dt}{dg(x_0)} = x_1$$

$$\Rightarrow x_0 - \frac{g(x_0)}{g'(x_0)} = x_1$$

2b.) nach dem $\varepsilon$ Kriterium

$\Delta x < \varepsilon$; d.h. $x_{n+1} - x_n < \varepsilon$

$\varepsilon$ frei wählbar

Prüfung 06.02.2007 Teil Nagele

1a.) $$g(x) = g(x_0) + \frac{1}{1!} \cdot g'(x_0)\,\Delta x + \frac{1}{2!}\,g''(x_0)\,\Delta x^2 + \ldots$$

$$x_{k+1} = x_k + \Delta x \Rightarrow \Delta x = x_{k+1} - x_k$$

$$\Rightarrow g(x_{k+1}) = g(x_k) + \frac{1}{1!} \cdot g'(x_k) \cdot (x_{k+1} - x_k) + \frac{1}{2!}\,g''(x_k)\,(x_{k+1} - x_k)^2$$

1b.) $$g(x_{k+1}) = g(x_k) + g'(x_k) \cdot (x_{k+1} - x_k)$$

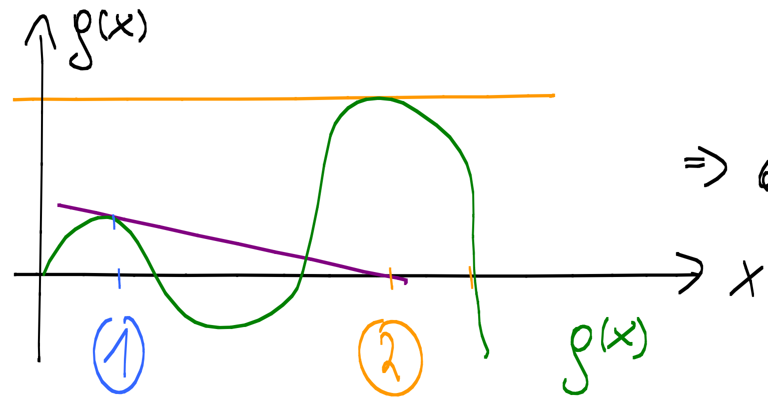
$$g(x_k) + g'(x_k) \cdot (x_{k+1} - x_k) \overset{!}{=} 0$$

$$x_{k+1} - x_k = -\frac{g(x_k)}{g'(x_k)}$$

$$x_{k+1} = x_k - \frac{g(x_k)}{g'(x_k)}$$

1c.) mit Hilfe des $\varepsilon$-Kriteriums;
$\varepsilon \Rightarrow$ frei wählbare Genauigkeit; Abbruch bei $x_{k+1} - x_k < \varepsilon$

1d.) nein, es kann divergieren




=> divergiert

Erfolg ist nur vom Startpunkt abhängig, Genauigkeit vom Wahl von $\varepsilon$

1e.) $u_R = i \cdot R$

$m_1: \quad U_= = u_R + u_{R,nl}$

$$i \cdot R + U_0 \left(\frac{i}{I_0}\right)^p - U_= = 0 = h(i)$$

1f.) $h'(i) = R + U_0 \frac{i^{p-1}}{I_0^p} (p)$

$$i_1 = i_0 - \frac{h(i)}{h'(i)}$$

$$h(1) = 5 + 10 \cdot \left(\frac{1}{0,15}\right)^{\frac{2}{3}} - 20 \quad | \; h(2) =$$

$$= 20,422$$

$$h(1)' = 28,615 \qquad | \; h'(2) =$$

$i_1 = 0,2863 \, A$

$i_2 = 0,3642 \, A$

$\Delta i = 0,7791 \, A$

1g.) $h(5) = 108{,}5744\,A$

$h'(5) = 18{,}8099\,A$

$i_1 = -0{,}7722\,A$

⇒ ein neg. Strom bedeutet das der eingezeichnete Strom in die andere Richtung fließt ⇒ Strom und Spannung an der Quelle laufen sind Phasengleich, das gibt es nicht.

⇒ math. dis. das Verfahren

2a.) $f(u,i) = u - i^2 = 0 \quad \Rightarrow \quad u = i^2$

Addition:

$$u_1 = i_1^2$$
$$u_2 = i_2^2$$
$$u_{ges} = u_1 + u_2 \overset{!}{=} (i_1 + i_2)^2$$
$$= i_1^2 + 2 i_1 i_2 + i_2^2 \neq i_1^2 + i_2^2$$

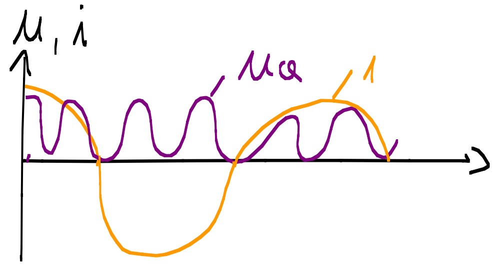
Homogen:

$$u \cdot a \overset{!}{=} (i \cdot a)^2 = i^2 \cdot a^2 \neq u \cdot a$$

⇒ Bauteil ist nicht linear da ein lineares Bauteil homogen & additiv ist.

2b.) $u = (\hat{I} \cdot \cos(\omega t))^2$

$= \hat{I}^2 \cdot \cos^2(\omega t)$




⇒ Frequenzverdopplung; ua nur pos.