Lineare algebra

Note Title 07.11.2007

Beispiel: $F: \mathbb{R}^2 \to \mathbb{R}$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \to a x_1 + b x_2 \qquad a, b \in \mathbb{R} \text{ und fix}$$

$$A = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}, \quad B = \{1\}$$

$$F\begin{pmatrix} 1 \\ 0 \end{pmatrix} = a 1 + b \cdot 0 = a = a \cdot 1$$

$$F\begin{pmatrix} 0 \\ 1 \end{pmatrix} = a \cdot 0 + b \cdot 1 = b = b \cdot 1$$

$$M_A^B(F) = \begin{pmatrix} a & b \end{pmatrix} \in M(1 \times 2)$$

$$v \in \mathbb{R}^2: \quad v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$x = c_A(v) = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$y = c_B(F(v)) = M_B^A(F) \cdot x = (a; b) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = a x_1 + b x_2$$

Beispiel: $P_1 \to P_2 \qquad v(t) \to (t+1) \cdot v(t)$ ist linear

$$A = \{1, 1-t\}$$

$$B = \{1+t, 1-t, t^2\}$$

$$F(a+bt) = (t+1)(a+bt) = a + bt + at + bt^2$$

$$F(1) = 1+t = 1 \cdot (1-t) + 0(1-t) + 0(t^2) = a + bt + at + bt^2$$

$$F(1-t) = 1-t^2 = \tfrac{1}{2}(1+t) + \tfrac{1}{2}(1-t) - 1(t^2) = a + t(a+b) - t^2 \cdot b$$

$$M_B^A(F) = \begin{pmatrix} 1 & 1/2 \\ 0 & 1/2 \\ 0 & -1 \end{pmatrix}$$

Beispiel: $F: \mathbb{P}_3 \to \mathbb{P}_4$

$$v(t) \to \int_0^t v(t)\,dt$$

$$F(a+bt+ct^2+dt^3) = \int_0^t (a+bt+ct^2+dt^3)\,dt$$

$$= at + b\frac{t^2}{2} + c\frac{t^3}{3} + d\frac{t^4}{4}\Big|_0^t$$

$$= at + b\frac{t^2}{2} + c\frac{t^3}{3} + d\frac{t^4}{4}\Big|_0^t$$

$A = \{1, t, t^2, t^3\}$ $\quad F(1) = t$

$B = \{1, t, t^2, t^3, t^4\}$ $\quad F(t) = \frac{t^2}{2}$

$F(t^2) = \frac{t^3}{3}$

$F(t^3) = \frac{t^4}{4}$

$$F(1) = 0 \cdot 1 + 1 \cdot t + 0t^2 + 0t^3 + 0t^4$$

$$F(t) = 0 \cdot 1 + 0t + \frac{1}{2}t^2 + 0t^3 + 0t^4$$

$$F(t^2) = 0 \cdot 1 + 0 \cdot t + 0t^2 + \frac{1}{3}t^3 + 0t^4$$

$$F(t^3) = 0 \cdot 1 + 0t + 0t^2 + 0t^3 + \frac{1}{4}t^4$$

$$M^A_B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/4 \end{pmatrix}$$

ges.: $F(1-2t+3t^2-4t^3)$

$$x = c_A(v) = \begin{pmatrix} 1 \\ -2 \\ 3 \\ -4 \end{pmatrix}$$

$$y = c_B(F(v)) = M^A_B(F) \cdot x$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

$$F(v) = c_B^{-1}(y) = 0 \cdot 1 + 1 \cdot t + (-1)t^2 + 1t^3 + (-1)t^4$$

$$F(1-2t+3t^2-4t^3) = \int (1-2\tau+3\tau^2-4\tau^3)\,dt$$

$$= t - t^2 + t^3 - t^4$$

Beispiel: $F : \mathbb{P}_2 \to \mathbb{P}_3$

$$p(t) \to p(t+2)$$

$$F(a + b1 + c1^2) = a + b(t+2) + c(t+2)^2$$

$$\mathcal{A} = \{1, t+2, (t+2)^2\}$$

$$\mathcal{B} = \{1, t, t^2\}$$

Bilden der Basisvektoren:

$$F(1) = 1 = 1 = 1 \cdot 1 + 0 \cdot t + 0 t^2$$

$$F(t+2) = (t+2) + 2 = t + 4 = 4 \cdot 1 + 1 t + 0 t^2$$

$$F((t+2)^2) = ((t+2)+2)^2 = \underbrace{(t+4)^2}_{t^2 + 2 \cdot 4t + 4^2} = 16 \cdot 1 + 8t + 1 t^2$$

$$M_{\mathcal{B}}^{\mathcal{A}}(F) = \begin{pmatrix} 1 & 4 & 16 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{pmatrix}$$

ges.: $F(\underbrace{3+t}_{v}) = ?$

$$v = 3 + t = 1 \cdot 1 + 1(t+2) + 0(t+2)^2$$

$$x = c_{\mathcal{A}}(v) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$c_{\mathcal{B}}(F(v)) = y = M_{\mathcal{B}}^{\mathcal{A}}(F) \cdot x = \begin{pmatrix} 1 & 4 & 16 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}$$

$$F(v) = c_B^{-1}(y) = 5 \cdot 1 + 1 \cdot t + 0 t^2 = 5 + t$$

Direkt: $F(3+t) = 3 + (t+2) = 5 + t$

Bsp.: Nullabb. $0 : V \to W,\ v \to 0$

$$M_B^A(0) = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{pmatrix} \in M(m \times n)$$

$\dim V = n$

$\dim W = m$