linA Tut

Note Title 28.01.2008

$$A = \begin{pmatrix} -1 & 2 & 2 \end{pmatrix} \in M(1 \times 3)$$

$$A = U \Sigma V^T$$

$$A^T = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

$$A^T \cdot A = \begin{array}{c|ccc} & -1 & 2 & 2 \\ \hline -1 & 1 & -2 & -2 \\ 2 & -2 & 4 & 4 \\ 2 & -2 & 4 & 4 \end{array}$$

$$\det(A^T A - I\lambda)$$

$$\begin{vmatrix} 1-\lambda & -2 & -2 & 1-\lambda & -2 \\ -2 & 4-\lambda & 4 & -2 & 4-\lambda \\ 4 & 4 & 4-\lambda & 4 & 4 \end{vmatrix}$$

$$(1-\lambda)(4-\lambda)(4-\lambda) + (-2)(4)(4) + (-2)(-2)(4) -$$

$$4 \cdot (-2)(4-\lambda) - (4)(4)(1-\lambda) - (4-\lambda)(-2)(-2)$$

$$(1-\lambda)(16 - 8\lambda + \lambda^2) - 32 + 16 - (-32 + 8\lambda) - (16 - 16\lambda)$$

$$-(16 - 16\lambda)$$

$$\cancel{16} - \cancel{8\lambda} + \lambda^2 - \cancel{16\lambda} + 8\lambda^2 - \lambda^3 - \cancel{16} + \cancel{32} - \cancel{8\lambda}$$
$$-\cancel{16} + \cancel{16\lambda} - \cancel{16} + \cancel{16\lambda}$$

$$= -\lambda^3 + 9\lambda^2 = \lambda^2(-\lambda + 9) \overset{!}{=} 0$$

$$\lambda_1 = 9 \qquad \sigma_1 = \sqrt{\lambda_1} = 3$$
$$\lambda_2 = 0 \qquad \sigma_2 = 0$$
$$\lambda_3 = 0 \qquad \sigma_3 = 0$$

$$\begin{pmatrix} 1-\lambda & -2 & -2 \\ -2 & 4-\lambda & 4 \\ -2 & 4 & 4-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \overset{!}{=} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{matrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{matrix}$$

$$\lambda_1 = 9$$

$$-\frac{1}{4} \quad \begin{pmatrix} -8 & -2 & -2 \\ -2 & -5 & 4 \\ -2 & 4 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \overset{!}{=} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-\frac{1}{4}$$

$$(1) \quad \begin{pmatrix} -8 & -2 & -2 \\ 0 & -4\tfrac{1}{2} & 4\tfrac{1}{2} \\ 0 & 4\tfrac{1}{2} & -4\tfrac{1}{2} \end{pmatrix} \longmapsto \begin{pmatrix} -8 & -2 & -2 \\ 0 & -4\tfrac{1}{2} & 4\tfrac{1}{2} \\ 0 & 0 & 0 \end{pmatrix} \Big| : 4\tfrac{1}{2}$$

$$\begin{pmatrix} -8 & -2 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \Longrightarrow x_2 = x_3$$

$$-8x_1 - 2x_2 - 2x_3 = 0$$

$$-x_2 + x_3 = 0 \quad \Rightarrow x_2 = x_3 = t$$

$$t \in \mathbb{R}$$

$$-8x_1 - 2x_2 - 2x_2 = 0$$

$$-8x_1 - 4x_2 = 0$$

$$-8x_1 = 4x_2 \quad /:2$$

$$-2x_1 = x_2$$

$$x_1 = -\frac{1}{2}x_2$$

$$v_1 = t\begin{pmatrix} -\frac{1}{2} \\ 1 \\ 1 \end{pmatrix} = \frac{t}{2}\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

$$\lambda = 0$$

$$\begin{pmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{pmatrix} -1 \quad \rightsquigarrow \quad \begin{matrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ 0 & 0 & 0 \end{matrix} \; 2$$

$$\rightsquigarrow \begin{pmatrix} 1 & -2 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{matrix} x_1 = 2x_2 + 2x_3 \\ x_2 = s, \; x_3 = t \end{matrix}$$

$$v_{2,3} = \begin{pmatrix} 2s+2t \\ s \\ t \end{pmatrix} \quad s,t \in \mathbb{R}$$

$$v_2 \Big|_{\substack{t=0 \\ s=1}} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \qquad v_3 \Big|_{\substack{t=1 \\ s=0}} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

$$w_1 = \frac{1}{\|v_1\|} \cdot v_1 \qquad \|v_1\| = \sqrt{(-1)^2 + (2)^2 + (2^2)}$$

$$= \sqrt{1 + 4 + 4} = 3$$

$$w_1 = \frac{1}{3} \cdot \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

$$w_2 = \frac{1}{\|u_2\|} \cdot u_2 \qquad u_2 = v_2 - \sum_{k=1}^{1} \langle v_2, w_k \rangle \cdot w_k$$

$$= \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \left\langle \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} \right\rangle \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \underbrace{(-2+2)}_{0} \cdot \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

$$\|u_2\| = \sqrt{2^2+1} = \sqrt{5}$$

$$w_2 = \frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\\0\end{pmatrix}$$

$$w_3 = \frac{1}{\|u_3\|}\cdot u_3 \qquad u_3 = v_3 - \langle v_3, w_2\rangle \cdot w_2$$

$$u_3 = \begin{pmatrix}2\\0\\1\end{pmatrix} - \left\langle \begin{pmatrix}2\\0\\1\end{pmatrix}, \begin{pmatrix}2\\1\\0\end{pmatrix}\frac{1}{\sqrt{5}}\right\rangle \cdot \frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\\0\end{pmatrix}$$

$$= \begin{pmatrix}2\\0\\1\end{pmatrix} - \frac{4}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\\0\end{pmatrix}$$

$$= \begin{matrix}2\\0\\1\end{matrix} - \frac{4}{5}\begin{pmatrix}2\\1\\0\end{pmatrix}$$

$$= \begin{matrix}2-\frac{8}{5} = \\ -\,4/5 = \\ \end{matrix}\begin{pmatrix}2/5\\-4/5\\1\end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix}2\\-4\\5\end{pmatrix}$$

$$\|u_3\| = \frac{1}{5}\cdot\sqrt{2^2+(-4)^2+5^2} =$$

$$\sqrt{4+16+25}$$

$$\|u_3\| = \frac{1}{5} \cdot \sqrt{45}$$

$$w_3 = \frac{5}{\sqrt{45}} \cdot \begin{pmatrix} 2/5 \\ -4/5 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{45}} \begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$$

$$V = \begin{pmatrix} w_1 & w_2 & w_3 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{3} & \frac{2}{\sqrt{5}} & \frac{2}{\sqrt{45}} \\ \frac{2}{3} & \frac{1}{\sqrt{5}} & \frac{-4}{\sqrt{5}} \\ \frac{2}{3} & 0 & \frac{5}{\sqrt{45}} \end{pmatrix}$$

$$u = \frac{1}{\sigma_i} \cdot A \cdot v_i$$

$$= \frac{1}{3} \cdot \begin{pmatrix} -1 & 2 & 2 \end{pmatrix} \begin{pmatrix} -1/3 \\ 2/3 \\ 2/3 \end{pmatrix} = 1$$

$$= 1 \qquad D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \Sigma = \begin{pmatrix} 3 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & 2 \\ 2 & 2 & -2 \end{pmatrix}$$