

lin A - Tutorium

Note Title

24.10.2007

1.)

$$A = \begin{pmatrix} 1 & 2 & -2 & 1 \\ 2 & 4 & -3 & 1 \\ -1 & 2 & 1 & 1 \\ 3 & -3 & 2 & -7 \end{pmatrix} \begin{array}{l} \downarrow (-2) \\ \swarrow \cdot 1 \\ \searrow (-3) \end{array}$$

ges: $A = L \cdot R$
 $PA = L \cdot R$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & ? & 1 & 0 \\ 3 & ? & ? & 1 \end{pmatrix}$$

$$\rightarrow A = \begin{pmatrix} 1 & 2 & -2 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 4 & -1 & 2 \\ 0 & -9 & 8 & -10 \end{pmatrix} \begin{array}{l} \uparrow \\ \downarrow \end{array} \quad L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ -1 & ? & 1 & 0 \\ 2 & ? & ? & 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

auch in P und L vertauschen

$$A = \begin{pmatrix} 1 & 2 & -2 & 1 \\ 0 & -9 & 8 & -10 \\ 0 & 4 & -1 & 2 \\ 0 & 0 & 1 & -1 \end{pmatrix} \downarrow (+9/9)$$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ -1 & -9/9 & 1 & 0 \\ 2 & 0 & ? & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & -2 & 1 \\ 0 & -9 & 8 & -10 \\ 0 & 0 & \frac{23}{9} & -\frac{23}{9} \\ 0 & 0 & 1 & -1 \end{pmatrix} \downarrow (9/23)$$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ -1 & -\frac{9}{9} & 1 & 0 \\ 2 & 0 & \frac{9}{23} & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & -2 & 1 \\ 0 & -9 & 8 & -10 \\ 0 & 0 & \frac{23}{9} & -\frac{23}{9} \\ 0 & 0 & 0 & -\frac{1}{23} \end{pmatrix} = h$$

1b.) $\det(A) = ?$

$$\det(P) = \det(L) \cdot \det(R)$$

$$\det(P) \cdot \det(A) = 1 \cdot 1$$

$$\det(P) = -1$$

$$\det(A) = \det(L) \cdot \det(R) \cdot \underbrace{(-1)}$$

die Zeilen vertauscht

$$2.) \quad A = \begin{pmatrix} 0 & a & 1 \\ 1 & 2 & 0 \\ -4 & -4 & a \end{pmatrix}$$

ges.: für welche $a \in \mathbb{R} \exists A^{-1}$

$$\Rightarrow \exists A^{-1} : \det A \neq 0$$

$$\begin{pmatrix} 0 & a & 1 \\ 1 & 2 & 0 \\ -4 & -4 & a \end{pmatrix} \begin{matrix} 0 & a \\ 1 & 2 \\ -4 & -4 \end{matrix}$$

$$\begin{aligned} \det(A) &= 0 \cdot 2 \cdot a + a \cdot 0 \cdot (-4) + 1 \cdot 1 \cdot (-4) \\ &\quad - (-4)(2)1 - (-4) \cdot 0 \cdot 0 - a \cdot 1 \cdot a \\ &= -4 + 8 - a^2 = 4 - a^2 \end{aligned}$$

$$4 - a^2 \stackrel{!}{=} 0 \Rightarrow \begin{matrix} a_1 = 2 \\ a_2 = -2 \end{matrix}$$

$$\Rightarrow \exists A^{-1} \Leftrightarrow a \in \mathbb{R} \setminus \{-2, 2\}$$

$$3.) \quad A = \begin{pmatrix} 0 & 0 & 1 & 2 \\ -1 & 1 & 3 & 2 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix}$$

ges.: A^{-1}

$$(A|I) \Rightarrow (I|A^{-1})$$

$$\left(\begin{array}{cccc|cccc} 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 \\ -1 & 1 & 3 & 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 0 & 0 & 1 \end{array} \right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 3 & 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 \end{array} \right) \begin{array}{l} \downarrow (+2) \\ \downarrow (-1) \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2 & 2 & 2 & 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 \end{array} \right) \begin{array}{l} \downarrow (-1) \\ \downarrow (-1) \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2 & 2 & 2 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 \end{array} \right) \begin{array}{l} \downarrow (+1) \\ \downarrow (+1) \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2 & 2 & 2 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 \end{array} \right) \begin{array}{l} | :2 \\ \uparrow (+n) \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 0 & -1 & 2 & -2 & 2 \\ 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 \end{array} \right) \uparrow (-n)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & -1 & 3/2 & -1 & 3/2 \\ 0 & 0 & 1 & 0 & -1 & 2 & -2 & 2 \\ 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 \end{array} \right) \begin{array}{l} \uparrow (+n) \\ \uparrow (+n) \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & -1 & 2 & -2 & 3 \\ 0 & 1 & 0 & 0 & 0 & -1/2 & 1 & -1/2 \\ 0 & 0 & 1 & 0 & -1 & 2 & -2 & 2 \\ 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 \end{array} \right) \uparrow (-n)$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & 5/2 & -3 & 7/2 \\ 0 & 1 & 0 & 0 & 0 & -1/2 & 1 & -1/2 \\ 0 & 0 & 1 & 0 & -1 & 2 & -2 & 2 \\ 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 \end{array} \right) = (I | A^{-1})$$

$$LR^{-1} = Q = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix} \quad P = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Zuerst die Zeilenvertauschungen \rightarrow nur in P notwendig
dann die z in L eliminieren;