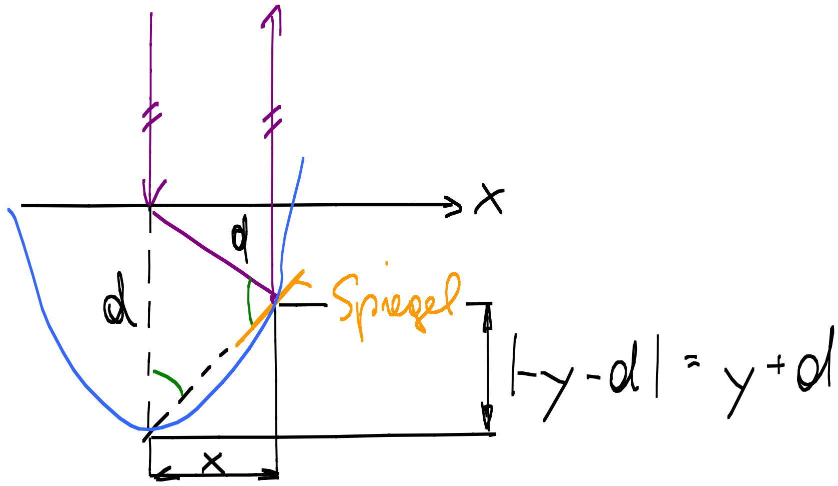
Dgl. Ue

8.)



(1) $y' = k = \frac{\Delta y}{\Delta x} = \frac{y + \sqrt{x^2+y^2}}{x}$ (Dgl.)

$y' = \frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2} = f\left(\frac{x}{y}\right)$

(2) $\left(\frac{y}{x}\right) = z \qquad z'x + z = y$

$z'x + z = z + \sqrt{1 + z^2}$

(3) $\int \frac{dz}{\sqrt{1+z^2}} = \int \frac{dx}{x}$

(4) $\ln\left|z + \sqrt{1 + z^2}\right| = \ln|x| + \ln C$

$z + \sqrt{1 + z^2} = x \cdot C$

$\frac{y + \sqrt{x^2 + y^2}}{x} = Cx$

$y + \sqrt{x^2+y^2} = C \cdot x^2 \qquad |-y \;|^2$

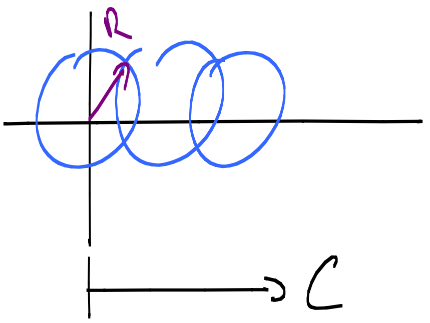
$$x^2 + \cancel{y^2} = C^2x^2 - 2Cx^2y + \cancel{y^2} \quad \Big| \, \frac{1}{x^2}$$

$$1 = C^2x^2 - 2Cy$$

(5) $$\Rightarrow y = \frac{C^2x^2 - 1}{2C}$$

$$\text{AWP: } C = \frac{1+\sqrt{2}}{2}$$

Q.)



R...const

$$\frac{\partial}{\partial x} \Big|$$

(1) $$\text{I: } (x-c)^2 + y^2 = R^2$$ ... Kreisgleichung

$$2(x-c) + 2yy' = 0$$

$$(x-c) = -yy'$$ $\Rightarrow$ in I einsetzen

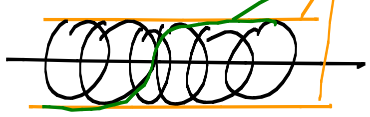
(2) $$(-yy')^2 + y^2 = R^2$$

$$y^2y'^2 + y^2 = R^2$$

$$y^2y'^2 = R^2 - y^2 \quad \Big| \cdot \frac{1}{y^2} \quad \Big| \sqrt{\ }$$

$$y' = \sqrt{\frac{R^2 - y^2}{y}}$$

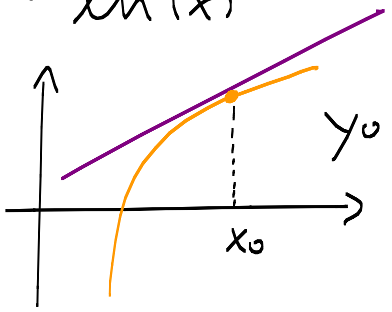
(3) Lösungen



Lösung nicht eindeutig

10.)

$$y = \ln(x) \qquad y - y_0 = y_0'(x - x_0)$$



$$y_0 = \ln(x_0)$$

$$y_0 = \ln(x_0)$$

$$y_0' = \frac{1}{x_0}$$

(1) I.) $y - \ln(x_0) = \frac{1}{x_0}(x - x_0)$ Parameter x_0

$\Rightarrow y' - 0 = \frac{1}{x_0} \Rightarrow x_0 = \frac{1}{y'}$ $\Rightarrow$ in I einsetzen

(2) $y - \ln\left(\frac{1}{y'}\right) = y'\left(x - \frac{1}{y'}\right)$

(3) $y = x \cdot y' - 1 - \ln(y')$ ... Clairautsche Dgl.

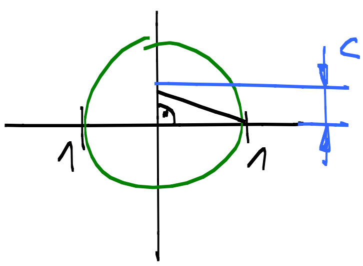
$\Rightarrow y = xy' + d(y')$

11.)

Kurvenschar mit Parameter

orthogonale Trajektorien

$y' \Leftrightarrow -\frac{1}{y'}$ $\Rightarrow$ Lösung



Gleichung der Kurvenschar:

(1) I.) $x^2 + (y-c)^2 = (1+c^2) \quad \Big| \frac{\partial}{\partial x}$

(2) $\cancel{2}x + \cancel{2}(y-c)\cdot y' = 0$

$x + (y-c)y' = 0$

$x + yy' = cy'$

$\frac{x}{y'} + y = c$ $\Rightarrow$ Einsetzen in I

(3) $\cancel{x^2 + \left(-\frac{x}{y'}\right)^2} = 1 + \cancel{\frac{x^2}{y'^2}} + 2\frac{xy}{y} + y^2$

$x^2 - y^2 - 1 = 2\frac{xy}{y'}$

$y' = \frac{2xy}{x^2-y^2-1}$ Dgl. der Kurvenschar

$y' \Leftrightarrow -\frac{1}{y'}$: $y' = -\frac{X^2-Y^2-1}{2XY}$ Dgl. der orthogonal. Trajektorien

(4) $(x^2-y^2-1)\,dx + 2xy\,dy = 0$ schreibe x statt X

$$\frac{N_x - M_y}{-N} = \frac{2y+2y}{-2xy} = -\frac{2}{x} = f(x)$$

(5) $\Rightarrow \mu(x) = e^{\int -\frac{2}{x}dx} = \frac{1}{x^2}$

(6) $\left(1-\frac{y^2}{x^2}-\frac{1}{x^2}\right)dx + \frac{2y}{x}\,dy = 0$ (jetzt exakte Dgl.)

(7) $\int_{x=1}^{x} 1-\frac{y^2}{x^2}-\frac{1}{x^2}\,dx + \int 2y\,dy = C$

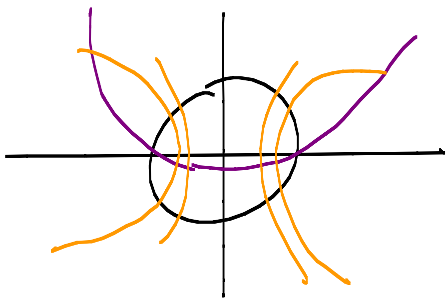
$\Rightarrow F(x,y) = \left| x + \frac{y^2}{x} + \frac{1}{x} \right|_1^x + y^2 = C$

$x + \frac{y^2}{x} + \frac{1}{x} - 1 - \frac{1}{y^2} - 1 + y^2 = C \quad |\cdot x$

$x^2 + y^2 + 1 = 2\tilde{C}x$

$x^2 - 2\tilde{C}x + \tilde{C} + y^2 = -1 + \tilde{C}^2$

$(x-\tilde{C})^2 + y^2 = \tilde{C}^2 - 1$ $\Rightarrow$ Kreisgl. mit Mittelpunkt auf x-Achse



11c.)

$x^2 + y^2 = c^2$

(1) $2x + 2yy' = 0 \Rightarrow y' = -\frac{x}{y}$ Dgl.

Isogonale Trajektorie: $y' \Leftrightarrow \frac{y' + \tan(\delta)}{1 - y'\tan(\delta)}$ $\delta = 45°$ $\tan(\delta) = 1$

$y' = \frac{-\frac{X}{Y} + 1}{1 - \left(-\frac{X}{Y}\right)\cdot 1} = \frac{-X+Y}{Y+X}$ Dgl. der orthogonalen Trajektorien

Lösung: $C\sqrt{1+\left(\frac{x}{y}\right)^2} = e^{-\arctan\frac{y}{x}}$

$$\sqrt{x^2+y^2} = \tilde{c}\, e^{-\arctan\frac{y}{x}}$$

Logarithmische Spirale: $r = \tilde{c}\cdot e^{-\varphi}$