Differentialgleichungen Rep

Note Title 11.03.2008

1.)

$$y' = \frac{x^2\sqrt{x^2+y^2}+y^2}{xy}$$

1 subst: $z = \frac{y}{x}$; prüfen ob möglich

$$y' = \frac{x^{\cancel{3}}\sqrt{1+\frac{y^2}{x^2}} + \left(\frac{y}{x}\right)^2 \cancel{x^2}}{\cancel{x^2}\,\frac{y}{x}} = f\left(x,\frac{y}{x}\right)$$

$$\left.\begin{aligned} zx &= y \\ z'x+z &= y' \end{aligned}\right\rbrace \frac{\partial}{\partial x}$$

2

$$z'x + z = \frac{x\sqrt{1+z^2}+z^2}{z} \quad \Big| -z$$

$$z'x = \frac{x\sqrt{1+z^2}}{z} + \cancel{\frac{z^2}{z}} - \cancel{z}$$

3 Trennung der Variablen

$$\int \frac{z\,dz}{\underbrace{\sqrt{1+z^2}}_{u}} = \int dx$$

$$2z = \frac{du}{dz} \Rightarrow 2z\,dz = du$$

$$\frac{1}{2}\int\frac{2z\,dz}{\sqrt{1+z^2}} = \frac{1}{2}\int\frac{du}{\sqrt{u}} = \frac{1}{\cancel{2}}\cancel{2}\sqrt{u}$$

$$= \sqrt{1+z^2} \quad = x + C$$

(4) Rücksubst:

$$\sqrt{1+\left(\frac{x}{y}\right)^2} = x + c \qquad \text{implizite Lösung}$$

$$1 + \left(\frac{x}{y}\right)^2 = (x+c)^2$$

$$\frac{y^2}{x^2} = (x+c)^2 - 1$$

$$y(x) = x \cdot \sqrt{(x+c)^2 - 1} \qquad \text{explizite Lösung}$$

2.) $$y' = \left(\frac{3x+y}{5x-y+8}\right)^2 = f\left(\frac{ax+by+c}{\alpha x+\beta y+\gamma}\right)$$

(1) $$\begin{vmatrix} a & b \\ \alpha & \beta \end{vmatrix} = \begin{vmatrix} 3 & 5 \\ 5 & -1 \end{vmatrix} = -8 \neq 0$$

$$x = \bar{x} + x_0 \qquad a x_0 + b y_0 = -c$$

$$y = \bar{y} + y_0 \quad \text{mit} \quad \alpha x_0 + \beta y_0 = -\gamma$$

(2)
$$\begin{aligned} 3x_0 + y_0 &= 0 \\ 5x_0 - y_0 &= -8 \\ \hline \Sigma\ 8x_0 &= -8 \end{aligned}$$

$$\Rightarrow x_0 = -1$$
$$\Rightarrow y_0 = 3$$

③ subst: $x = \bar{x} - 1$

$y = \bar{y} + 3$

$$\bar{y}' = \left(\frac{3\bar{x} \cancel{-3} + \bar{y} \cancel{+3}}{5\bar{x} \cancel{-5} - \bar{y} \cancel{-3} \cancel{+8}}\right)^2$$

$$= \left(\frac{3\bar{x} + \bar{y}}{5\bar{x} - \bar{y}}\right)^2$$

$$= \left(\frac{3 + \frac{\bar{y}}{\bar{x}}}{5 - \frac{\bar{y}}{\bar{x}}}\right)^2$$

④ $$\bar{z}'\bar{x} + \bar{z} = \left(\frac{3+\bar{z}}{5-\bar{z}}\right)^2 \quad \Big| -\bar{z}$$

$$\bar{z}'\bar{x} = \frac{9 + 6\bar{z} + \bar{z}^2 - 25\bar{z} + 10\bar{z}^2 - \bar{z}^3}{(5-\bar{z})^2}$$

$$= +z^3 - 11\bar{z}^2 + 19\bar{z} \underline{-9}$$

Die Zahl ist das Produkt der Nullstellen des Polynomes

$$= (+z^3 - 11\bar{z}^2 + 19\bar{z} - 9) : (\bar{z} - 1) = (\bar{z}^2 - 1)(z - 9)$$

$$-\bar{z}^3 + \bar{z}^2$$

$$10\bar{z}^2 + 10\bar{z}$$

$$-9\bar{z} + 9$$

$$0\ R$$

⑤ $$\int \frac{-(5-\bar{z})^2 \, dz}{(\bar{z}-1)^2(\bar{z}-9)} = \int \frac{d\bar{x}}{\bar{x}}$$

⑥ Partialbruchzerlegung:

$$\ldots = \frac{A}{\bar{z}-1} + \frac{B}{(\bar{z}-1)^2} + \frac{C}{(\bar{z}-9)}$$

$$-25 + 10\bar{z} - \bar{z}^2 = A(\bar{z}^2 - 10\bar{z} + 9) + B(\bar{z} - 9) + C(\bar{z}^2 - 2\bar{z} + 1)$$

Koeffizientenvergleich:

$$1: -25 = 9A + 9B + C$$

$$\bar{z}: 10 = -10A + B - 2C$$

$$\bar{z}^2: -1 = A + C$$

$$\Sigma \quad -16 = 0 - 8B + 0 \Rightarrow B = 2$$

$$\Rightarrow A = -\frac{3}{4}$$

$$C = -\frac{1}{4}$$

⑦ $$-\frac{3}{4}\int \frac{1}{\bar{z}-1}\,d\bar{z} + 2\int \frac{d\bar{z}}{(\bar{z}-1)^2} - \frac{1}{4}\int \frac{d\bar{z}}{\bar{z}-9} = \int \frac{d\bar{x}}{\bar{x}}$$

$$= -\frac{3}{4}\ln|\bar{z}-1| - 2\cdot\frac{1}{\bar{z}-1} - \frac{1}{4}\cdot\ln|z-9| = \ln|\bar{x}| + \ln|c|$$

⑧ $$= -\frac{3}{4}\ln\left|\frac{\bar{y}-\bar{x}}{\bar{x}}\right| - \frac{2\bar{x}}{\bar{y}-\bar{x}} - \frac{1}{4}\ln\left|\frac{\bar{y}-9\bar{x}}{\bar{x}}\right| = \ln|\bar{x}| + \ln|c|$$

$$= \frac{3}{4}\ln|\bar{y}-\bar{x}| + \cancel{\frac{3}{4}\ln|\bar{x}|} - \frac{2\bar{x}}{\bar{y}-\bar{x}} - \frac{1}{4}\ln|\bar{y}-9\bar{x}| + \cancel{\frac{1}{4}\ln|\bar{x}|}$$

$$= \cancel{\ln|\bar{x}|} + \ln|c| \qquad |\ \exp$$

(9)
$$\frac{\exp\left(-\frac{2\bar{x}}{\bar{y}-\bar{x}}\right)}{\sqrt[4]{(\bar{y}-\bar{x})^3\,(\bar{y}-9\bar{x})}} = C$$

(10) rücksubst:
$$\bar{x} = x+1$$
$$\bar{y} = y-3$$

$$\frac{\exp\left(\frac{-2(x+1)}{y-x-4}\right)}{\sqrt[4]{(y-x-4)^3\,(y-x-12)}} = C$$

3.)
$$3(y'-2)x + 2(y'-2)y = 5$$
$$y'[3x+2y] = 6x+4y+5$$

(1)
$$y' = \frac{6x+4y+5}{3x+2y}$$

(2)
$$\begin{vmatrix} 6 & 4 \\ 3 & 2 \end{vmatrix} = 12-12 = 0$$

(3) subst:
$$3x+2y = u$$
$$3+2y' = u' \quad \frac{d}{dx}$$
$$y' = \frac{u'-3}{2}$$

$$\Rightarrow \frac{u'-3}{2} = \frac{2u+5}{u} \quad | \cdot 2$$

$$u'-3 = \frac{4u+10}{u} \quad | + 3$$

$$u' = \frac{7u+10}{u}$$

4 $\int \frac{u\,du}{7u+10} = \int dx$

$$u : (7u+10) = \frac{1}{7} - \frac{10}{7} \cdot \frac{1}{7u+10}$$

$$-u \pm \frac{10}{7}$$

$$= \frac{1}{7}u - \frac{10}{7 \cdot 7} \cdot \ln|7u+10| = x + \tilde{c}$$

$$7u - 10 \ln|7u+10| = 49x + \tilde{c}$$

5 rücksubst:

$$21x + 14y - 10 \ln|21x+14y+10| = 49x + \tilde{c}$$

$$= 14y - 28x - \tilde{c} = 10 \ln|21x+14y+10|$$

alternativ über exakte Dgl:

$$\frac{dy}{dx} = y' = (6x + 4y + 5)dx - (3x + 2y)dy = 0$$

integrierender Faktor: $X(x) \cdot Y(y)$

$$\mu(x,y) = e^{\frac{19}{5}x} \cdot e^{-\frac{7}{5}y}$$