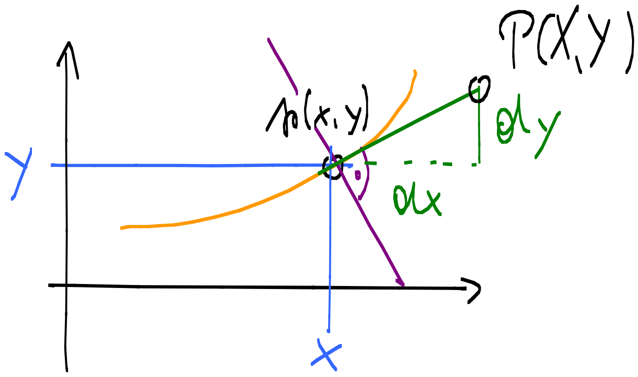
Differentialgleichungen VO

Note Title 12.03.2008

S. 28:



k ... Steigung

$$\frac{dy}{dx} = k = y'$$

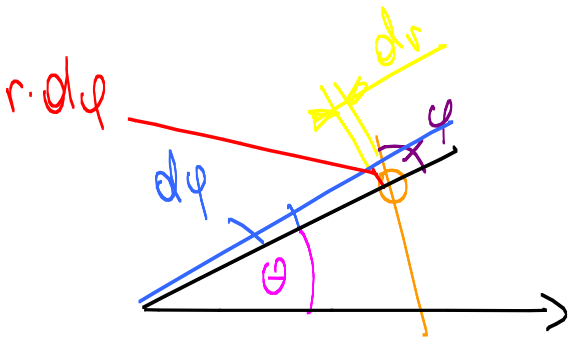
Tangente: $p(x,y)$ ... Werte sind beliebig aber fix

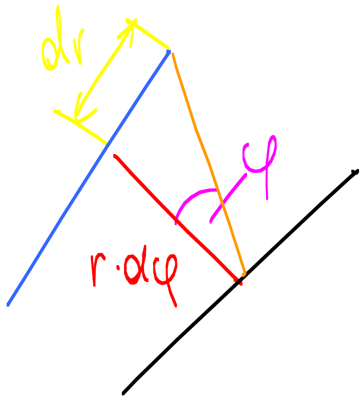
$$Y - y = y' \cdot (X - x)$$

normale auf Tangente:

$$k = -\frac{1}{y'(x,y)}$$

für Polarkoordinaten:





$$\tan\varphi = \frac{r\,d\theta}{dr} = \frac{r}{r'}$$

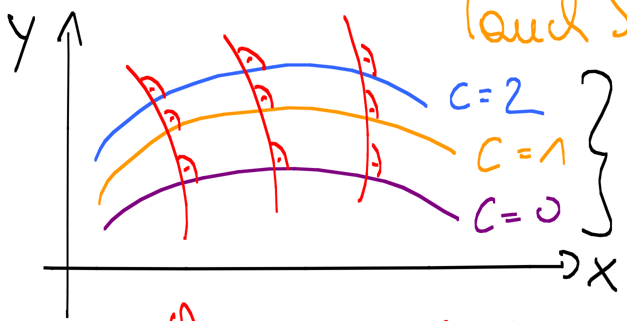
mit $r' = \dfrac{dr}{d\theta}$

Trajektorien:

- Dgl $f(x,y,y')=0$ 1 Gleichung

$\Rightarrow$ Lösung $F(x,y)=C$ n Lösungen (Kurvenschar)

Parameter (auch Scharparameter)



$f(x,y,y')=0$

Orthogonale Trajektorie $F(X,Y,Y')=0$

$$\Rightarrow \quad X=x;\; Y=y;\; Y'\neq y'$$

$$y' \Leftrightarrow -\frac{1}{Y'};\quad Y'=-\frac{1}{y'}$$

Beispiel:

Kurvenschar: $y=(x+c)^3$ / Ableiten

$$y'=3(x+c)^2\cdot 1$$

aus diesen 2 Gleichungen das C - Eliminieren

$$y^{1/3}=x+c$$

$y'=3(y^{1/3})^2$ ... parameterfreie Dgl

$$(x, y, y') \Leftrightarrow (X, Y, Y' = \frac{1}{y'})$$

$$-\frac{1}{y'} = 3(y^{1/3})^2$$ Dgl der orthogonalen Trajektori

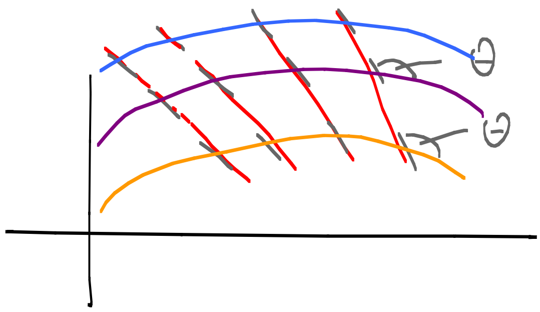
$$y' = \frac{1}{3} y^{-\frac{2}{3}}$$

$$\int dy \cdot y^{\frac{2}{3}} = \int \frac{1}{3} dx$$

$$\frac{y^{\frac{5}{3}}}{\frac{5}{3}} = -\frac{1}{3} x + c$$

$$y = -\frac{5}{9} x + \frac{5}{3} c \ldots$$ Schargleichung der orthogonalen Trajektorie

Isogonale Trajektorien mit Winkel Θ



$$Y' \Leftrightarrow \frac{y' + \tan\Theta}{1 - y'\tan\Theta}$$

für $\Theta = \frac{\pi}{2} = 90°$ erhält man die orthogonalen

Beispiel:

Gerade mit Anstieg $30°$

$$y = \frac{1}{\sqrt{3}} x + C$$

$$y' = \frac{1}{\sqrt{x}}$$ ... parameterfrei

Berechne isogonale Trajektorie mit $\Theta = 30°$

$$\tan \Theta = \frac{1}{\sqrt{3}}$$

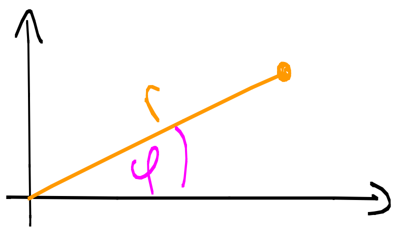
$$y' = \frac{y' + \frac{1}{\sqrt{3}}}{1 - y' \frac{1}{\sqrt{3}}} = \frac{\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}}$$

$$y' = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$ ... isogonal

$$y = \sqrt{3}\, x + C$$

Orthogonale Trajektorie in Polarkoordinaten:

$$r = r(\varphi)$$



$$\text{Steigung} = \frac{r}{r'} \Leftrightarrow -\frac{R'}{R}$$

$$r = R$$
$$\varphi = \phi$$

$$R' = \frac{-r\,R}{r'} = \frac{-r^2}{r'}$$

Kurve in Polarkoordinaten:

$$(r, \varphi, r') = 0 \Leftrightarrow \left(R, \phi, R' = -\frac{r^2}{r'}\right)$$