Datenstrukturen & Algorithmen Tutorium:

Note Title 22.10.2007

1.) $n! \stackrel{?}{=} O(n^n)$

$c = 1$ $\quad 5! \leq 1 \cdot 5^5$

$n_0 = 5$ $\quad 120 \leq 3125$

$n(n-1)(n-2)\cdots\cdots 1 \leq \underbrace{n \cdot \cdots n}_{n}$

2.) $\ln(n) = O(n)$

$\lim\limits_{n\to\infty} \frac{\ln(n)}{\sqrt{n}}$ De l' Hospital

$\rightarrow \frac{\frac{1}{n}}{\frac{1}{2\sqrt{n}}} \rightarrow 0$

3.) $\ln(n!) \stackrel{?}{=} O(n\ln(n))$

$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$

$\lim\limits_{n\to\infty} \frac{\ln\left(\sqrt{2\pi n}\left(\frac{n}{e}\right)^n\right)}{n\ln(n)} \rightarrow$ De l' Hospital

$\lim\limits_{n\to\infty} \frac{\sqrt{2\pi n}\,\frac{1}{\sqrt{2\pi n}}\left(\frac{n}{e}\right)^n + \sqrt{n}\left(\frac{n}{e}\right)^n \left(\ln\left(\frac{n}{e}\right)+1\right)}{\sqrt{2\pi}\sqrt{n}\left(\frac{n}{e}\right)^n\left(\ln(n)+1\right)}$

$\lim\limits_{n\to\infty} \frac{\frac{1}{\sqrt{2\pi n}} + \sqrt{n}\left(\ln\left(\frac{n}{e}\right)+1\right)}{\sqrt{n}\left(\ln(n)+1\right)}$

$$\lim_{n\to\infty} \frac{1+2n\ln(n)}{2n+2n\ln(n)}$$ De l' Hospital

$$\lim_{n\to\infty} \frac{2+2\ln(n)}{4+2\ln(n)}$$

$$= \lim_{n\to\infty} \frac{2n}{2n} = 1 < \infty$$

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alternative Lösung

$$\ln(n!) = \Theta(n\ln(n)) \Leftrightarrow \exists c_1, c_2 : c_1 \ln(n!) \leq \ln n^n < c_2 \ln(n!)$$

$$e^{c_1 \ln(n!)} \leq e^{\ln(n^n)} \leq e^{c_2 \ln(n!)}$$

$$(n!)^{c_1} \leq n^n \leq (n!)^{c_2}$$

$$c_2 = 2 :\Rightarrow_{2.2} n^n \leq (n!)^2$$

$$(n^n) = \prod_{k=0}^{n-1} n$$

$$(n!) = \prod_{k=0}^{n-1} (n-k)(k+1)$$

$$\prod_{k=0}^{n-1} n \leq \prod_{k=0}^{n-1} (n-k)(k+1)$$

$$n-1 \geq k \quad / \cdot k \quad \Rightarrow nk \geq k^2+k$$

$$n+nk \geq n+k+k^2$$

$$n+nk-k-k^2 \geq n$$

$$(n-k)(k+1) \geq n$$

$$\Rightarrow n^n \leq (n!)^2$$

zu $\prod_{k=0}^{n-1}(n-k)(k+1) = (n!)^2$

$n \cdot n \cdot (n-1)(n-2) \cdots\cdots 2 \cdot 2 \cdot 1 \cdot 1$

$\underbrace{(n \cdot 1)}_{(n-k)}(n-1)(2) \cdots\cdots (n-2)(2)\underbrace{(n \cdot 1)}_{k+1}$

4.) $f(n) = \Theta(g(n)) \Rightarrow 2^{f(n)} = \Theta(2^{g(n)})$

z.B.: $f(n) = 2n$

$g(n) = n$

$\lim_{n\to\infty} \frac{2n}{n} = 2 < \infty$

$\lim_{n\to\infty} \frac{2^{2n}}{2^{n}} = 2^{n} = \infty$