

$$1.) \quad \frac{dx}{dt} = A \cdot x$$

$$\frac{dx_1}{dt} = -4x_1 + 6x_2$$

$$\frac{dx_2}{dt} = -1x_1 + 1x_2$$

$$2.) \quad \frac{dx_1}{dt} = -4x_1 + 6x_2 + 0u$$

$$\frac{dx_2}{dt} = -1x_1 + 1x_2 + 1u$$

$$y = \begin{bmatrix} 1 \\ -1 \end{bmatrix} x + u$$

$$1x_1 - x_2 + u$$

$$2c.) \quad G(s) = \frac{y(s)}{u(s)} \Big|_{x_0=0} = c^T (sI - A)^{-1} \cdot b + d$$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}^T \begin{bmatrix} s+4 & 6 \\ -1 & s-1 \end{bmatrix}^{-1} b + d$$

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \frac{1}{(s+4)(s-1) + 6} \begin{bmatrix} s-1 & +6 \\ -1 & s+4 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} + 1$$

$$\left[\begin{array}{cc|c} & & 0 \\ & & 1 \\ \hline S-1 & +6 & 6 \\ -1 & S+4 & S+4 \end{array} \right]$$

$$[1 \ -1] \frac{1}{s^2+3s+2} \begin{bmatrix} 6 \\ S+4 \end{bmatrix} + 1$$

$$\left[\begin{array}{cc|c} & & 6 \\ & & S+4 \\ \hline 1 & -1 & 6+(-S-4) \end{array} \right] + 1$$

$$\frac{2-s + 1(s^2+3s+2)}{s^2+3s+2} = \frac{s^2 + 2s + 4}{s^2 + 3s + 2}$$

$$\begin{array}{ccc} 1 & 2 & 4 \\ 1 & 3 & 2 \end{array}$$

$$3.) G(s) = \frac{Y(s)}{U(s)} \Big|_{x_0=0} = c^T (sI - A)^{-1} b + d$$

$$[1 \ -1] \begin{bmatrix} S+4 & -6 \\ -1 & S+1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1$$

$$[1 \ -1] \frac{1}{(S+4)(S+1) - 6} \begin{bmatrix} S+1 & +6 \\ +1 & S+4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1$$

$$\begin{array}{c|c} & \begin{array}{c} +6 \\ s+4 \end{array} \\ \hline [1 & -1] \end{array} \begin{array}{c} +6 - s - 4 \end{array}$$

$$\begin{array}{cc|c} & & 0 \\ & & 1 \\ \hline s+1 & +6 & +6 \\ 1 & s+4 & s+4 \end{array}$$

$$= \frac{-(s+2) + (s^2 + 5s + 2)}{s^2 + 5s - 2}$$

$$= \frac{s^2 + 4s + 0}{s^2 + 5s - 2}$$