(48) (a) $f(x) = \frac{P(x)}{Q(x)} = \frac{x+1}{(x^2+4)(x^2+9)}\, dx$

Grad $Q \geq$ Grad $P+2$, Q keine Nullst. in $\mathbb{R}$ $\rightarrow$ Bem. 42, S. 90

$$\int_{-\infty}^{+\infty} f(x)\, dx = 2\pi i \sum_{\operatorname{Im} z_j > 0} \operatorname{Res}(f(z), z_j)$$

$z_1 = 2i \quad z_2 = -2i \quad z_3 = 3i \quad z_4 = -3i \qquad \operatorname{Im} z_2, z_4 < 0$

$$\operatorname{Res}(f, 2i) = \left.\frac{z+1}{(z+2i)(z^2+9)}\right|_{z=2i} = \frac{2i+1}{4i(+5)} = \frac{2i+1}{20i}$$

$$= +\frac{1}{10} - \frac{i}{20}$$

$$\operatorname{Res}(f, 3i) = \left.\frac{z+1}{(z^2+4)(z+3i)}\right|_{z=3i} = \frac{3i+1}{(-5)\,6i} = \frac{-3i-1}{30i}$$

$$= \frac{i}{30} - \frac{1}{10}$$

$$\int_{-\infty}^{+\infty} f(x)\, dx = 2\pi i\left(-\frac{i}{60}\right) = \underline{\frac{\pi}{30}}$$

(b) $f(x) = \frac{\cos x}{x^2+6x+10} = \frac{1}{2}\left(\underbrace{\frac{e^{ix}}{x^2+6x+10}}_{=:g(x)} + \underbrace{\frac{e^{-ix}}{x^2+6x+10}}_{=:h(x)}\right)$ Bem. 44, S. 92

${}_1z_2 = -3 \pm \sqrt{9-10} = -3 \pm i \qquad \operatorname{Im} z_1 > 0 \quad \operatorname{Im} z_2 < 0 \rightarrow$

$$\operatorname{Res}(g(z), -3+i) = \left.\frac{e^{iz}}{z-(-3-i)}\right|_{z=-3+i} = \frac{+e^{-1-3i}}{2i}$$

$$\operatorname{Res}(h(z), -3-i) = \left.\frac{e^{-iz}}{z-(-3+i)}\right|_{z=-3-i} = \frac{-e^{-1+3i}}{2i}$$

$$\int_{-\infty}^{+\infty} f(x)\, dx = \frac{1}{2} \cdot 2\pi i \cdot \left\{\operatorname{Res}(g(z), -3+i) - \operatorname{Res}(h(z), -3-i)\right\}$$

$$= \pi i \cdot e^{-1}\left\{\frac{1}{2i}e^{-3i} + \frac{1}{2i}e^{+3i}\right\}$$

$$= \frac{\pi}{2e}\left\{e^{-3i} + e^{+3i}\right\}$$

$$= \frac{\pi}{2e}\left\{+2\cos 3\right\}$$

$$= \underline{\frac{\pi \cdot \cos 3}{e}}$$

(48c) $f(x) = \frac{P(x)}{Q(x)} = \frac{x^2}{(x^2+2x+2)^3}\,dx$ Bem. 42

${}_1z_2 = -1 \pm \sqrt{1-2} = -1 \pm i$, je 3fache Nullst. , keine reelle Nullst. &

Grad $Q \geq$ Grad $P + 2$ $\rightarrow$

$$\mathrm{Res}(f(z), -1+i) = \lim_{z\to(-1+i)} \frac{1}{2!}\frac{d^2}{dz^2}\left\{(z-(-1+i))^3 \frac{z^2}{(z^2+2z+2)^3}\right\}$$

$$= \frac{1}{2}\lim_{z\to(-1+i)} \frac{d^2}{dz^2}\frac{z^2}{(z+1+i)^3}$$

$$\cancel{= \frac{1}{2}\cdot\frac{2z(z+1+i)^{3}-3(z+1+i)^2\cdot z^2}{(z+1+i)^{4}}}\Big|_{z=-1+i}$$

$$\cancel{= \frac{1}{2}\cdot\frac{2(-1+i)\cdot 2i - 3\cdot(-1+i)^2}{(2i)^4}}$$

$(i-1)^2 = -1-2i+1 = -2i$

$$\cancel{= \frac{1}{2^5}\cdot(-4i-4+6i)}$$

$(-1+i)(1+i) = -1+i^2 = -2$

$$\cancel{= \frac{i-2}{2^4}}$$

$$\cancel{\int_{-\infty}^{+\infty} f(x)\,dx = 2\pi i\cdot\sum_{\mathrm{Im}\, z_j>0}\mathrm{Res}(f(z), z_j)}$$

$$= \frac{1}{2}\cdot\lim_{z\to(-1+i)}\frac{d}{dz}\frac{2z(z+1+i)-3z^2}{(z+1+i)^4}$$

$$= \quad \ldots \quad \frac{-z^2+2z(1+i)}{(z+1+i)^4}$$

$$= \frac{1}{2}\,\frac{(-2z+2+2i)(z+1+i)-4(-z^2+2z(1+i))}{(z+1+i)^5}\Big|_{z=-1+i}$$

$$= \frac{1}{2}\,\frac{2(-z+1+i)(z+1+i)-4(-z^2+2z(1+i))}{(z+1+i)^5}\Big|_{z=-1+i}$$

$$= \frac{1}{2}\,\frac{2\cdot 2\cdot 2i - 4\cdot(+2i-4)}{(2i)^5}$$

$$= \frac{1}{2^6}\cdot\frac{16}{i} = \underline{\frac{1}{4i}}$$

$$\int_{-\infty}^{+\infty} f(x)\,dx = 2\pi i\cdot\sum_{\mathrm{Im}\, z_j>0}\mathrm{Res}(f(z), z_j) = 2\pi i\cdot\frac{1}{4i}$$

$$= \underline{\frac{\pi}{2}}$$