(44) Konvergenzradius: $R = \frac{1}{\limsup \sqrt[n]{|a_n|}}$, $R = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right|$ f. $\sum_{n=0}^{\infty} a_n (z-z_0)^n$

(a) $\sum_{n=0}^{\infty} \frac{n^n}{n!} z^n$: $a_n = \frac{n^n}{n!}$ ~~$\left|\frac{a_n}{a_{n+1}}\right| = \left(\frac{n}{n+1}\right)^{n} \frac{(n+1)\,n!}{n!} = \frac{(n+1)}{(n+1)} \frac{1}{(1+\frac{1}{n})^n}$~~

$\underline{R} = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| = \lim \left| \frac{n^n}{(n+1)^n \cdot (n+1)} \cdot \frac{(n+1)\,n!}{n!} \right| = \lim \frac{1}{(1+\frac{1}{n})^n}$

$\underline{= \frac{1}{e}}$

(b) $\sum_{n=0}^{\infty} z^{n^2} = \sum_{n=0}^{\infty} (z^n)^n = \sum_{n=0}^{\infty} (z^{n-1})^n \cdot z^n$

$a_n = (z^{n-1})^n \qquad a_{n+1} = (z^n)^{n+1} \qquad \left|\frac{a_n}{a_{n+1}}\right| = z^{-2n} \xrightarrow{n\to\infty} \begin{cases} \infty & z<1 \\ 1 & z=1 \\ 0 & z>1 \end{cases}$

$\to$ konvergiert für $\underline{|z| \leq 1}$

(c) $\sum_{n=1}^{\infty} \frac{1}{n^2} \cdot z^n$ $\qquad$ $\sum_{n=1}^{\infty}$ hat gl. Konv.r. wie $\sum_{n=0}^{\infty}$ weil nur um Konst. versch.

$\underline{R} = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| = \lim \frac{n^2}{n^2+2n+1} = \lim \frac{1}{1+\frac{2}{n}+\frac{1}{n^2}} = 1$

$\underline{= 1}$

(b) $\sum_{n=0}^{\infty} z^{n^2}$ $\to \sup|a_n| = 1, \ \sup \sqrt[n]{|a_n|} = 1$

$\to R = \frac{1}{\limsup \sqrt[n]{|a_n|}} \ \underline{= 1}$

$\sum_{n=0}^{\infty} 1 \cdot z^{n^2} = 1 \cdot z^0 + 1 \cdot z^1 + 0 \cdot z^2 + 0 \cdot z^3 + 1 \cdot z^4$