(42) $u_{tt} = 4u_{xx} \quad \to a = \frac{1}{4}$

$$u(x,t) = \sum_{k=1}^{\infty}\left(c_k \cos\left(\frac{k\pi}{L\sqrt{a}}t\right) + d_k \sin\left(\frac{k\pi}{L\sqrt{a}}\cdot t\right)\right)\cdot \sin\left(\frac{k\pi}{L}\cdot x\right)$$

NB: (1) $u(0,t) = u(L,t) = 0$

(2) $u(x,0) = \frac{L}{2} - |x-\frac{L}{2}| = \begin{cases} x & 0\le x\le L/2 \\ L-x & L/2 \le x \le L\end{cases} \quad = f(x)$

(3) $u_t(x,0) = -x^2 + Lx \quad = g(x)$

$$c_k = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{k\pi}{L}x\right)dx$$

$$= \frac{2}{L}\left\{\int_0^{L/2} x\sin\left(\frac{k\pi}{L}x\right)dx + \int_{L/2}^{L}(L-x)\sin\left(\frac{k\pi}{L}x\right)dx\right\} \qquad \int fg' = fg - \int f'g$$

$$= \frac{2}{L}\left\{\left(-\frac{L}{k\pi}\right)x\cos\left(\frac{k\pi}{L}x\right)\Big|_0^{L/2} + \int_0^{L/2}\frac{L}{k\pi}\cos\left(\frac{k\pi}{L}x\right)dx\right.$$

$$\left.-L\cdot\frac{L}{k\pi}\cos\left(\frac{k\pi}{L}x\right)\Big|_{L/2}^{L} + x\cdot\frac{L}{k\pi}\cos\left(\frac{k\pi}{L}x\right)\Big|_{L/2}^{L} - \int_{L/2}^{L}\frac{L}{k\pi}\cos\left(\frac{k\pi}{L}\cdot x\right)dx\right\}$$

$$= \frac{2}{k\pi}\left\{-\cancel{\frac{L}{2}\cos\left(\frac{k\pi}{2}\right)} + \frac{L}{k\pi}\sin\left(\frac{k\pi}{L}x\right)\Big|_0^{L/2}\right.$$

$$\left.-L\cdot\left(\cancel{\cos(k\pi)} - \cancel{\cos\left(\frac{k\pi}{2}\right)}\right) + L\cdot\left(\cancel{\cos(k\pi)} - \cancel{\frac{\cos\left(\frac{k\pi}{2}\right)}{2}}\right) - \frac{L}{k\pi}\sin\left(\frac{k\pi}{L}x\right)\Big|_{L/2}^{L}\right\}$$

$$= \frac{2L}{k\pi}\left\{\frac{2}{k\pi}\sin\left(\frac{k\pi}{2}\right) - \cancel{2\cos(k\pi)} + \cancel{\cos\left(\frac{k\pi}{2}\right)} + \cancel{\frac{1}{k\pi}\sin\left(\frac{k\pi}{2}\right)}\right\} = \frac{4L}{(k\pi)^2}\sin\left(\frac{k\pi}{2}\right)$$

$$= \begin{cases} 0 & k = 2n \\ 4L/(k\pi)^2\cdot(-1)^n & k = 2n+1\end{cases}$$

$$d_k = \frac{2\sqrt{a}}{k\pi}\int_0^L(-x^2+Lx)\sin\left(\frac{k\pi}{L}x\right)dx \qquad \text{siehe (41)}$$

$$= \frac{1}{k\pi}\cdot\frac{4L^2}{(k\pi)^3}\left(1-(-1)^k\right) = \begin{cases} 8L^2/(k\pi)^4 & \forall k\in N_u \quad k = 2n+1,\ n = 0,1,\dots \\ 0 & \forall k \in N_g\end{cases}$$

$$u(x,t) = \sum_{n=0}^{\infty}\left\{\frac{4L(-1)^n}{((2n+1)\pi)^2}\left(\cos\left(\frac{(2n+1)\pi}{2L}\cdot t\right) + \frac{8L^2}{((2n+1)\pi)^4}\sin\left(\frac{(2n+1)\pi}{2L}\cdot t\right)\right)\right\}\cdot\sin\left(\frac{(2n+1)\pi}{L}\cdot x\right)$$