(41) $u_{tt} = c^2 u_{xx} \quad \rightarrow \quad a = \frac{1}{c^2}$

allg. Lösg.:

$$u(x,t) = \sum_{k=1}^{\infty} \left(c_k \cos\left(\frac{k\pi}{L\sqrt{a}} \cdot t\right) + d_k \sin\left(\frac{k\pi}{L\sqrt{a}} \cdot t\right)\right) \cdot \sin\left(\frac{k\pi}{L} \cdot x\right)$$

NB: (1) $u(0,t) = u(L,t) = 0$

(2) $u(x,0) = x\cdot(L-x) \quad \rightarrow \quad f(x) = -x^2 - Lx$

(3) $u_t(x,0) = 0 \quad \rightarrow \quad g(x) = 0$

betr. halbes Periodenintervall $[0,L) \rightarrow$ Bem. 31 S. 68 $\xrightarrow{\cos\text{-Koeff.}=0}$

$$c_k = \frac{2}{L}\int_0^L f(x) \sin\left(\frac{k\pi}{L}x\right) dx$$

$$d_k \cdot \frac{k\pi}{L\sqrt{a}} = \frac{2}{L}\int_0^L g(x) \sin\left(\frac{k\pi}{L}x\right) dx \xrightarrow{g(x)=0} d_k = 0 \;\; \forall k$$

$$c_k = \frac{2}{L}\int_0^L (-x^2 + Lx) \sin\left(\frac{k\pi}{L}x\right) dx \qquad \int fg' = fg - \int f'g$$

$$= \frac{2}{L}\left\{ \underbrace{(-x^2 + Lx)\left(-\frac{L}{k\pi}\right)\cos\left(\frac{k\pi}{L}x\right)\Big|_0^L}_{\to 0} + \int_0^L (-2x+L)\frac{L}{k\pi}\cdot\cos\left(\frac{k\pi}{L}x\right) dx \right\}$$

$$= \frac{2}{L}\left\{ L\cdot\left(\frac{L}{k\pi}\right)^2 \underbrace{\sin\left(\frac{k\pi}{L}\cdot x\right)}_{\to 0}\Big|_0^L - 2x\left(\frac{L}{k\pi}\right)^2 \underbrace{\sin\left(\frac{k\pi}{L}x\right)\Big|_0^L}_{\to 0} + 2\left(\frac{L}{k\pi}\right)^2 \int_0^L \sin\left(\frac{k\pi}{L}x\right) dx \right\}$$

$$= -\frac{2}{L}\cdot 2\cdot\left(\frac{L}{k\pi}\right)^3 \cdot \cos\left(\frac{k\pi}{L}\cdot x\right)\Big|_0^L$$

$$= \frac{4L^2}{(k\pi)^3}\cdot\left(1 - (-1)^k\right) = \begin{cases} 8L^2/(k\pi)^3 & \forall k \in \mathbb{N}_u \\ 0 & \forall k \in \mathbb{N}_g \end{cases}$$

$$u(x,t) = \sum_{k=0}^{\infty} \frac{8L^2}{((2k+1)\pi)^3} \cdot \cos\left(\frac{(2k+1)\pi c}{L}\cdot t\right) \cdot \sin\left(\frac{(2k+1)\pi}{L}\cdot x\right)$$