Analysis T2 Übung 10

Note Title 13.06.2007

45.) z → arctan:

$$x = \tan(\underline{z}) = \frac{\sin(\underline{z})}{\cos(\underline{z})} = \frac{\frac{e^{jz}-e^{-jz}}{2j}}{\frac{e^{jz}+e^{-jz}}{2}}$$

$$x = \frac{e^{jz}-e^{-jz}}{j(e^{jz}+e^{-jz})} \quad | \cdot j(\ )$$

$$jx(e^{jz}+e^{-jz}) = e^{jz}-e^{-jz}$$

$$jx\,e^{jz} + \frac{jx}{e^{jz}} = e^{jz} - \frac{1}{e^{jz}} \quad | \cdot e^{jz}$$

$$jx(e^{jz})^2 + jx = (e^{jz})^2 - 1$$

$$jx(e^{jz})^2 - (e^{jz})^2 = -1 - jx$$

$$(e^{jz})^2(jx-1) = -(1+jx)$$

$$e^{jz} = \sqrt{\frac{+(1+jx)}{+(1-jx)}} \quad | \ln$$

$$jz = \frac{1}{2} \ln\left(\frac{1+jx}{1-jx}\right) \quad | : j$$

$$\underline{\underline{z = \frac{1}{2j} \ln\left(\frac{1+jx}{1-jx}\right)}}$$

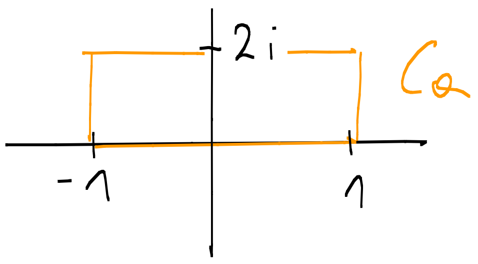
$$e^{jx} = \cos(x) + j\sin(x)$$

$$e^{-jx} = \cos(x) - j\sin(x)$$

$$e^{jx} + e^{-jx} = 2\cos(x)$$

$$e^{jx} - e^{-jx} = j2\sin(x)$$

46.) a.)



$$\oint_{C_a} \frac{z^2+1}{(z-i)^2}\,dz$$

CIS: $f^{(k)}(z_0) = k! \cdot \frac{1}{2\pi j} \cdot \oint \frac{f(z)}{(z-z_0)^{k+1}}\,dz$

$k=1$; $z_0 = j$

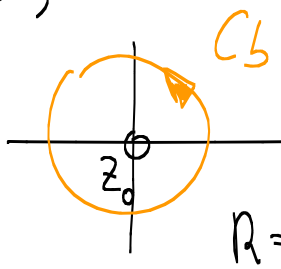
$$f'(j) = \frac{1}{2\pi j} \cdot \oint_{C_a} \frac{z^2+1}{(z-j)^2}\,dz$$

$f'(z) = 2z \qquad f'(j) = 2j$

$$2j = \frac{1}{j2\pi} \oint \frac{z^2+1}{(z-1)^2}\, dz$$

$$\oint \frac{z^2+1}{(z-1)^2}\, dz = -4\pi$$

46b.)



$R=2$

$$\oint \frac{z^2 e^z}{(z+1)^3}\, dz$$

$$f'(z) = 2z e^z + z^2 e^z$$

$$f''(z) = 2e^z + 2ze^z + 2ze^z + z^2 e^z$$

$$f''(z) = e^z(2 + 4z + z^2)$$

$R=2$

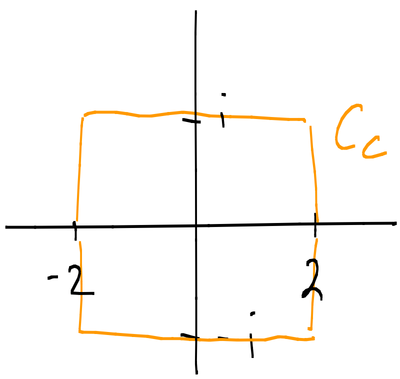
$z_0 = -1$

$$f''(-1) = \frac{1-4+2}{e} = -\frac{1}{e}$$

$$f''(-1) = \frac{2}{2j\pi} \oint \frac{z^2 e^z}{(z-z_0)^3}\, dz$$

$$\oint \frac{z^2 e^z}{(z-z_0)^3}\, dz = \frac{-j\pi}{e}$$

46c.)



$$\oint_{C_c} \frac{z-2}{(z+1)(z+3)}\, dz$$

49.) $$z^{37} - 3z^{26} + 8z^{17} - 19z^{10} - 4z^4 + 2$$

$h(z)$ $\qquad\qquad$ $g(z)$

$|z| < 1$ ... Einheitskreis

$$|g(z)| \leq 19 \cdot |1|^{10} = 19$$

$$|h(z)| \leq 1 + 3 + 8 + 4 + 2 = 18$$

$$|h(z)| \leq |g(z)| \quad \text{auf } |z| < 1$$

Satz von Rouché $\Rightarrow$ $f(z)$ hat gleich viele Nullstellen innerhalb Einheitskreis wie $g(z)$, also 10

=

48.) a.) $$\int_0^{2\pi} \frac{3 + 4\sin(x)}{5 + 3\cos(x)}\, dx$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2} = \frac{1}{2}\left(z + \frac{1}{z}\right)$$

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j} = \frac{1}{2j}\left(z - \frac{1}{z}\right)$$

$$e^{jx} := \underline{z} \qquad \frac{dz}{dx} = je^{jx} = jz \qquad dx = \frac{dz}{jz}$$

$$= \oint_{|z|=1} \frac{3 + \frac{4}{2j}\left(z - \frac{1}{z}\right)}{5 + \frac{3}{2}\left(z - \frac{1}{z}\right)} \frac{dz}{jz} \qquad \Big| \cdot 2j$$

$$= \oint_{|z|=1} \frac{6j + 4z - \frac{4}{z}}{10j + 3j\left(z - \frac{1}{z}\right)} \frac{dz}{j}$$

$$= \oint_{|z|=1} \frac{6j + 4z - \frac{4}{z}}{-10 - 3z + \frac{3}{z}}$$

$$= 2\pi j \cdot \sum \operatorname{Res} \frac{6j + 4z - \frac{4}{z}}{-10 - 3z + \frac{3}{z}} \Big|_{z}$$