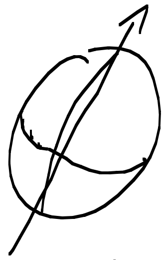
Analysis Übungen 07

Notiztitel 09.05.2007

30 a.) $(x^2+y^2+z^2)^2 = xyz$



er 0, daher mehrere Bereiche

Kugel-Koord.?

$$\begin{cases} x = r\cos\varphi\sin\delta \\ y = r\sin\varphi\sin\delta \\ z = r\cos\delta \end{cases}$$

$$(r^2\cos^2\varphi\sin^2\delta + r^2\sin^2\varphi\sin^2\delta + r^2\cos^2\varphi)^2 = r^3$$

$$(r^2(\sin^2\delta(\cos^2+\sin^2) + \cos^2\delta))^2$$

$$r^{\cancel{4}} = \cancel{r^3}\sin^2\delta\cos\varphi\sin\varphi\cos\delta$$

$$\int_{\delta=0}^{\pi/2}\left[\int_{\varphi=0}^{\pi/2}(\sin^2\delta\cos\varphi\sin\varphi\cos\delta)\,d\varphi + \int_{\varphi=\pi}^{\varphi=\frac{3\pi}{2}}\sin^2\delta\cos\varphi\sin\varphi\cos\delta\,d\varphi\right]d\delta$$

$\pi/2$

$$+\int_{\pi/2}^{\pi}\left[\int_{\varphi=\pi/2}^{\pi}(\sin^2\delta\cos\varphi\sin\varphi\cos\delta)\,d\varphi + \int_{\varphi=\frac{3\pi}{2}}^{\varphi=2\pi}\sin^2\delta\cos\varphi\sin\varphi\cos\delta\,d\varphi\right]$$

$$= -\int_{\delta=0}^{\pi/2}\sin^2\delta\cos\delta\left(-\frac{\cos^2(\varphi)}{2}\Big|_0^{\pi/2} - \frac{\cos^2(\varphi)}{2}\Big|_{\pi}^{\frac{3\pi}{2}}\right)d\delta$$

$$+\int_{\delta=\frac{\pi}{2}}^{\delta=\pi}\sin^2\delta\cos\delta\left(\frac{\cos^2(\varphi)}{2}\Big|_{\frac{\pi}{2}}^{\pi} - \frac{\cos^2(\varphi)}{2}\Big|_{\frac{3\pi}{2}}^{2\pi}\right)d\delta$$

$$= \int_0^{\pi/2} \sin^2\delta \cos\delta \left(\frac{1}{2}+\frac{1}{2}\right) d\delta + \int_{\pi/2}^{\pi} \sin^2\delta \cos\delta \left(-\frac{1}{2}-\frac{1}{2}\right) d\delta$$

$$= \frac{\sin^3\delta}{3}\Big|_0^{\pi/2} - \frac{\sin^3\delta}{3}\Big|_{\pi/2}^{\pi} = \frac{1}{3}+\frac{1}{3} = \frac{2}{3}$$

30 b.) $x-y+z=6 \Rightarrow z = 6-x+y$

$x+y=2 \Rightarrow x = 2-y$

$0 \le y \le 1$

$$\int_{y=0}^{1}\int_{x=y}^{2-y}\int_{z=0}^{6-x+y} 1 \, dz\, dx\, dy$$

$$= \int_0^1 \int_{x=y}^{2-y} 6-x+y \; dx\, dy$$

$$= \int_0^1 \left(6x - \frac{x^2}{2} + xy \Big|_y^{2-y}\right) dy$$

$$= \int_0^1 \left(6(2-y) - \frac{(2-y)^2}{2} + (2-y)y - \left(6y - \frac{y^2}{2} + y^2\right)\right) dy$$

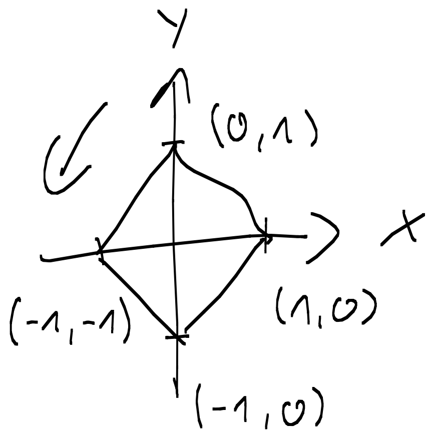
$$= \int_0^1 \left(12 - 6y - \frac{4-4y+\cancel{y^2}}{2} + 2y - y^2 - 6y + \frac{\cancel{y^2}}{\cancel{2}} - y^2\right) dy$$

$$= \int_0^1 (10 - 8y - 2y^2)\, dy$$

$$= \left(10y - \frac{8y^2}{2} - \frac{2y^3}{3}\right) \Big|_0^1$$

$$= 10 - 4 - \frac{2}{3} = \frac{16}{3}$$

31.) $\int_C (x-y)\, dx + (x^2+y^2)\, dy$ $\qquad |x|+|y| \leq 1$



Von $(1,0)$ auf $(0,1)$

$$\vec{x}(t) = t \cdot \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] = \begin{pmatrix} -t \\ +t \end{pmatrix}$$

$$\frac{dx}{dt} = -1 \qquad \frac{dy}{dt} = 1$$

$$dx = -dt \qquad dy = dt$$

$$\int_0^1 (-t - t)(-dt) + ((-t)^2 + (t)^2)\, dt$$

$$\int_0^1 (2t + 2t^2)\, dt = t^2 + \frac{2}{3} t^3 \Big|_0^1 = 1 + \frac{2}{3} = \frac{5}{3}$$

Von $(0,1)$ auf $(-1,0)$

$$\vec{x}(t) = t \left(\begin{pmatrix} -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} -t \\ -t \end{pmatrix}$$

$$dx = -dt$$
$$dy = -dt$$

$$\int_0^1 (-t+t)(-dt) + ((-t)^2 + (-t)^2)(-dt)$$

$$= \int_0^1 0 \quad - 2t^2$$

$$= -\frac{2}{3}t^3 \Big|_0^1 = -\frac{2}{3}$$

Von $(-1,0)$ auf $(0,-1)$

$$\vec{x}(t) = t \cdot \left(\begin{pmatrix} 0 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} t \\ -t \end{pmatrix}$$

$$dx = dt \qquad dy = -dt$$

$$\int_0^1 (t-(-t))\,dt + (t^2 + (-t)^2)(-dt)$$

$$= \int_0^1 (2t \quad - 2t^2)\,dt$$

$$= t^2 - \frac{2}{3}t^2 \Big|_0^1 = 1 - \frac{2}{3} = \frac{1}{3}$$

Von $(0,-1)$ auf $(1,0)$

$$\vec{x}(t) = t \cdot \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right) = \begin{pmatrix} t \\ t \end{pmatrix}$$

$$dx = dt \qquad dy = dt$$

$$\int_0^1 (t - t)\,dt + (t^2 + t^2)\,dt$$

$$= \int_0^1 0 + 2t^2\,dt$$

$$= \int_0^1 \frac{2}{3}t^2 \Big|_0^1 = \frac{2}{3}$$

$$\Rightarrow \int_C = \frac{5}{3} - \frac{2}{3} + \frac{1}{3} + \frac{2}{3} = \frac{6}{3} = 2$$

32.) $P_1 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \qquad P_2 = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$

$$\int_C x^2\,dx + xy\,dy + (1 + x + z)\,dz$$

$$\vec{x}(t) = t \cdot \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} t \\ t \\ -2t \end{pmatrix}$$

$$dx = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$\int_0^1 t^2 \, dt + t \cdot t \, dt + (1 + t - 2t)(-2dt)$$

$$= \int_0^1 \frac{t^3}{3} + \frac{t^3}{3} + \left(-2t - \frac{2t^2}{2} + \frac{4t^2}{2} \right) \Big|_0^1$$

$$\frac{2t^3}{3} - 2t + t^2 \Big|_0^1 = \frac{2}{3} - 2 + 1 = \frac{2}{3} - 1 = -\frac{1}{3}$$