Analysis

Notiztitel 26.04.2007

24.) $\iint_B f(x,y)\,dx\,dy$

$$f(x,y) = \sin(x+y) - y$$

$$B = [0,\tfrac{\pi}{2}] \times [0,\tfrac{\pi}{2}]$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\int_0^{\frac{\pi}{2}} \left(\int_0^{\frac{\pi}{2}} (\sin x \cos y + \cos x \sin y - y)\,dx \right) dy$$

$$= \int_0^{\pi/2} (\sin x \sin y + \sin(x)\sin(y) - yx)\,dy$$

$$= \int_0^{\pi/2} (2(\sin(x)\sin(y)) - yx \Big|_0^{\frac{\pi}{2}} \,dy$$

$$= \int_0^{\pi/2} (2 \underbrace{\sin(\tfrac{\pi}{2})}_{1} \sin(y) - y \cdot \tfrac{\pi}{2} - 0)\,dy$$

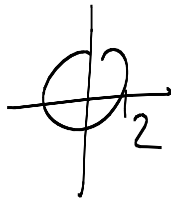
$$\int_0^{\pi/2} (2 \sin(y) - \frac{y\pi}{2})\,dy$$

$$\int_0^{\pi/2} (-2\cos(y) - \frac{y^2 \pi}{4} \Big|_0^{\frac{\pi}{2}}$$

$$= (\underbrace{-2(\cos\tfrac{\pi}{2}}_{0}) - \frac{\frac{\pi^2}{4}\pi}{4}) - (-2\cos(0))$$

$$= -\frac{\pi^3}{16} + 2$$

27.) $f(x,y) = (x+y)^2 \qquad B = \text{Kreis}$



$$\begin{array}{l|l} x = r\cos\varphi & 0 \le r \le 2 \\ y = r\sin\varphi & 0 \le \varphi \le 2\pi \end{array}$$

$$(x+y)^2 = (r\cos\varphi + r\sin\varphi)^2 = r^2(\cos\varphi + \sin\varphi)^2$$

$$= r^2(\cos^2\varphi + 2\sin\varphi\cos\varphi + \sin^2\varphi)$$

$$= r^2(1 + 2\sin\varphi\cos\varphi)$$

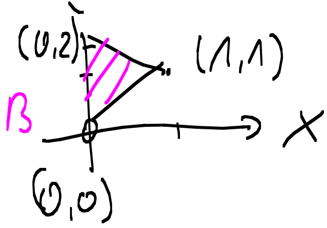
$$\det\left(\frac{\partial(x,y)}{\partial(r,\varphi)}\right) = \det\begin{pmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{pmatrix} = r(\cos^2 + \sin^2)$$

$$\int_0^2 \left(\int_0^{2\pi} r^2(1 + 2\sin\varphi\cos\varphi)\, r\, dr\, d\varphi \right.$$

$$\int_0^2 r^3\, dr \cdot \int_0^{2\pi} (1 + 2\sin\varphi\cos\varphi)\, d\varphi$$

$$\int_0^1 \frac{r^4}{4}\Big|_0^2 \cdot \int_0^{2\pi} \varphi - 2\cos\varphi\sin\varphi \Big|_0^{2\pi}$$

$$\frac{2^{\cancel{4}\,3}}{\cancel{4}} \cdot \cancel{2}\pi = 8\pi$$

$26.)\quad f(x,y) = x^2 y \qquad B =$ 

$$0 \le x \le 1$$
$$x \le y \le 2-x$$

$$\iint_B f(x,y) = \int_0^1 \left(\int_x^{2-x} (x^2 y)\, dy \right) dx$$

$$= \int_0^1 \left(\frac{x^2 y^2}{2} \Big|_0^{2-x} \right) dx$$

$$= \int_0^1 \left(\frac{x^2 (2-x)^2}{2} - \frac{x^2 (x)^2}{2} \right) dx$$

$$= \int_0^1 \left(\frac{x^2 \cdot (x^2 - 4x + 4)}{2} - \frac{x^4}{2} \right)$$

$$= \int_0^1 (-2x^3 + 2x^2)\, dx$$

$$= \int_0^1 \left(-\frac{\cancel{2}x^4}{2\cancel{4}} + \frac{2x^3}{3} \right) \Big|_0^1$$

$$= -\frac{1^4}{2} + \frac{2 \cdot 1^3}{3} = -\frac{1}{2} + \frac{2}{3} = \frac{-3+4}{6} = \frac{1}{6}$$

28.) $f(x,y) = x^2 y \qquad B = \{(x,y) \in \mathbb{R}^2 \mid 0 \leq x \leq 4,\ y^2 \leq x\}$

$$B = \quad 0 \leq x \leq 4$$
$$-\sqrt{x} \leq |y| \leq \sqrt{x}$$

$$\iint_B f(x,y) = \int_0^4 \left(\int_{-\sqrt{x}}^{\sqrt{x}} x^2 y \, dy \right) dx$$

$$= \int_0^4 \left(\int_{-\sqrt{x}}^{\sqrt{x}} \frac{x^2 y^2}{2} \Big|_{-\sqrt{x}}^{\sqrt{x}} \right) dx \qquad (-\sqrt{x})^2 = x$$

$$= \int_0^4 \left(\frac{x^2 x}{2} - \frac{x^2 x}{2} \right) = 0$$

29.) $f(x,y) = y^2 \quad B \quad xy = 16$

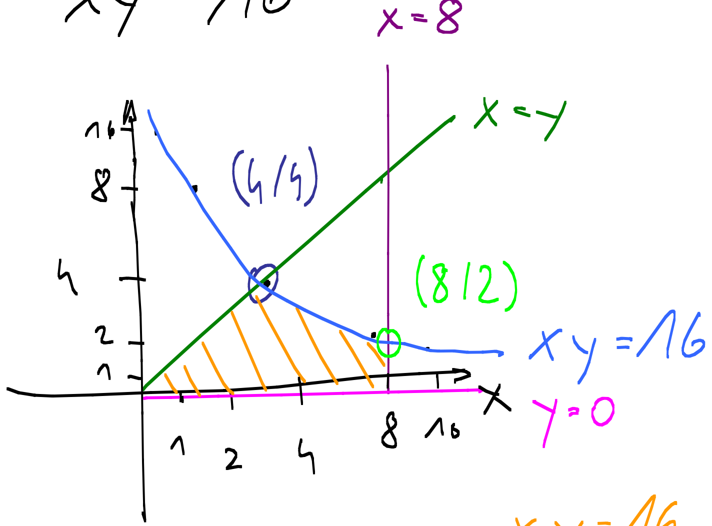
$x \cdot y = 16 \ / \ x = y$

$x^2 = 16$

$x = \pm 4 \rightarrow$ nur pos

$x \cdot y = 16 \ / \ x = 8$

$y = 2$



$x \cdot y = 16 \Rightarrow y = \frac{16}{x}$

$B_1 = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 4,\ 0 \leq y \leq x\}$

$B_2 = \{(x,y) \in \mathbb{R}^2 : 4 \leq x \leq 8,\ 0 \leq y \leq \frac{16}{x}\}$

$$\iint_B f(x,y) = \iint_{B_1} f(x,y) + \iint_{B_2} f(x,y)$$

$$= \int_0^4 \left(\int_0^x y^2 \, dy \right) dx) + \int_4^8 \left(\int_0^{16/x} y^2 \, dy \right) dx$$

$$= \int_0^4 \left(\int_0^x \frac{y^3}{3} \Big|_0^x \, dx \right) + \int_4^8 \left(\int_0^{16/x} \frac{y^3}{3} \Big|_0^{\frac{16}{x}} \, dx \right)$$

$$= \int_0^4 \left(\frac{x^3}{3} - 0 \right) dx + \int_4^8 \left(\frac{16^3}{x^3 3} - 0 \right) dx$$

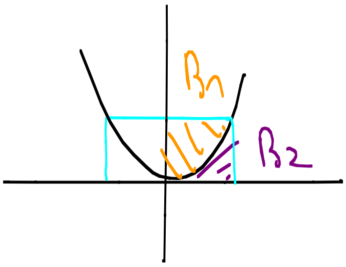
$$= \frac{x^4}{12} \Big|_0^4 + \frac{16^3 \cdot}{3 \cdot x^2 \cdot 2} \Big|_4^8$$

$$\frac{4^{\cancel{4}3}}{3 \cdot \cancel{4}} + \frac{16^3}{6 \cdot x^2}$$

$$\frac{4^3}{3} + \frac{16^3}{6} \left(\frac{1}{2^6} - \frac{1}{2^4} \right)$$

$$\frac{4^3 + 16^3 (2^{-6} - 2^{-4})}{3} = \frac{160}{3}$$

25.) $f(x,y) = \sqrt{|y - x^2|}$ $\qquad B = [-1,1] \times [0,1]$



$y \geq x^2$

$f(x,y) = \sqrt{y-x^2}$

$$\iint_B f(x,y) = 2 \cdot \iint_{B_1} + \iint_{B_2}$$

$x^2 \leq y$

$f(x,y) = \sqrt{x^2-y}$

$$= 2 \cdot \left[\int_0^1 \left(\int_{x^2}^1 \sqrt{y-x^2}\, dy \right) dx + \int_0^1 \left(\int_0^{x^2} \sqrt{x^2-y}\, dy \right) dx \right]$$

$$= 2 \left[\int_0^1 \left(\int_{x^2}^1 \frac{2}{3} \cdot (y-x^2)^{\frac{3}{2}} \Big|_{x^2}^{1} \right) dx + \int_0^1 \left(\frac{2}{3} (x^3-y)^{\frac{3}{2}} \Big|_0^{x^2} dx \right] \right.$$

$$= 2 \left[\underbrace{\frac{2}{3} \int_0^1 (1-x^2)^{\frac{3}{2}}\, dx}_{} + \frac{2}{3} \int_0^1 (x^2)^{\frac{3}{2}}\, dx = \frac{1}{6} \right]$$

$T_1: \frac{2}{3} \cdot \frac{3\pi}{16}$

$$= \frac{\pi}{4} + \frac{1}{3}$$