Übungszettel 2

Notiztitel 06.03.2007

6.) $d(x,y) = \begin{cases} \|x\|_2 + \|y\|_2 & x \neq y \\ 0 & \text{sonst} \end{cases}$

Metrik: M1.) $\cdot\ \|x\|_2 + \|y\|_2 = \underbrace{\sqrt{x^2}}_{\geq 0} + \underbrace{\sqrt{y^2}}_{\geq 0} \geq 0$ ✓

$\cdot\ 0 + 0 \geq 0$

M2.) $\cdot\ \|x\|_2 + \|y\|_2 = 0 \Rightarrow x = y = 0$?

$\cdot\ 0 + 0 = 0 \Rightarrow x = y = 0$?

M3.) $d(x,y) = d(y,x) = \cdot\ \|x\|_2 + \|y\|_2 = \|y\|_2 + \|x\|_2$ ✓

$\cdot\ 0 + 0 = 0 + 0$ ✓

$\sqrt{x^2} + \sqrt{y^2} = \sqrt{y^2} + \sqrt{x^2}$ ✓

M4.) $d(x,z) \leq d(x,y) + d(y,z)$

1. Fall $x=y$: $\|x\|_2 + \|z\|_2 \leq \|x\|_2 + \|y\|_2 + \|y\|_2 + \|z\|_2$

$\sqrt{x^2} + \cancel{\sqrt{z^2}} \leq 0 + \sqrt{y^2} + \cancel{\sqrt{z^2}}$ ✓

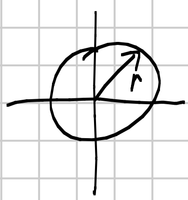
2. Fall $x=z$: $0 \leq \underbrace{\sqrt{x^2}}_{\geq 0} + \underbrace{\sqrt{y^2}}_{\geq 0} + \underbrace{\sqrt{y^2}}_{\geq 0} + \underbrace{\sqrt{z^2}}_{\geq 0}$ ✓

3. Fall $y=z$: $\cancel{\sqrt{x^2}} + \sqrt{z^2} \leq \cancel{\sqrt{x^2}} + \sqrt{y^2} + 0$ ✓

Kugel: $B(0,r) : \{ y \in X : \overbrace{\|x\|_2 + \|y\|_2}^{d(x,y)} < r \}$

$\|\vec{y}\|_2 < r - \|\vec{x}\|_2$

$\sqrt{y^2} < r - \sqrt{x^2}$



7.)

$$d(a,b) = \begin{cases} \frac{1}{\min(n \in \mathbb{N} \mid a(n) \neq b(n))} & a \neq b \\ 0 & \text{sonst} \end{cases}$$

$a, b \in \{0,1\}$ $a: \mathbb{N} \to \{0,1\}$ / $0 \notin \mathbb{N}$

M1.) $d(a,b) \geq 0$

• $a = b$: $0 \geq 0$ ✓

• $a \neq b$: $\frac{1}{\min(a(n) \neq b(n))} = 1 \geq 0$ ✓

True = 1, False = 0 ? ($a \neq b$: True)

M2.) • $a = b$: $0 = 0 \Rightarrow a = b$ ✓

• $a \neq b$: $\frac{1}{\min(a(n) \neq b(n))} = 0 \Rightarrow \frac{1}{0}$! Frage

M3.) • $a = b$: $0 = 0$ ✓

• $a \neq b$: $\frac{1}{\min(a(n) \neq b(n))} = \frac{1}{\min(b(n) \neq a(n))}$ ✓

M4.) $d(a,c) \leq d(a,b) + d(b,c)$

✓ 1 Fall: $a = c$: $0 \leq \frac{1}{\min(a(n) \neq b(n))} + \frac{1}{\min(b(n) \neq c(n))}$

✓ 2. Fall: $a = b$ $\frac{1}{\min(a(n) \neq c(n))} \leq 0 + \frac{1}{(\min(b(n) \neq c(n))}$

3. Fall: $b = c$ $\frac{1}{\min(a(n) \neq c(n))} \leq \frac{1}{\min(a(n) \neq b(n))} + 0$

8.) $f: \mathbb{R}^2 \to \mathbb{R}$

a.) $$f(x,y) = \begin{cases} \dfrac{x^2-y^2}{x^2+xy+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Polar: $$f(r\cdot\cos\varphi, r\cdot\sin\varphi) = \frac{\cancel{r^2}\cos^2\varphi - \cancel{r^2}\sin^2\varphi}{\cancel{r^2}\cos^2\varphi + \cancel{r^2}\cos\varphi\sin\varphi + \cancel{r^2}\sin^2\varphi}$$

$$\lim_{r\to 0}\left(\frac{1}{\underbrace{\cos^2\varphi + \cos\varphi\sin\varphi + \sin^2\varphi}_{\text{beschränkt durch } 3}}\right) \neq 0$$

Funktion unstetig!

b.) $$f(x,y) = \begin{cases} \dfrac{x^3+y^3}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = 0 \end{cases}$$

Polar: $$f(r\cos\varphi, r\sin\varphi) = \frac{\cancel{r^3}\cos^3\varphi + \cancel{r^3}\sin^3\varphi}{\cancel{r^2}\cos^2\varphi + \cancel{r^2}\sin^2\varphi}$$

$$= |r| \cdot \frac{\cos^3\varphi + \sin^3\varphi}{\cos^2\varphi + \sin^2\varphi} \leq$$

beschränkt

$\Rightarrow$ stetig

9a.)

$$f(x,y) = \begin{cases} \dfrac{xy^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$$f(r\cos\varphi, r\sin\varphi) = \frac{r \cdot \cos\varphi \cdot \cancel{r^2} \sin^2\varphi}{\cancel{r^2} \cos^2\varphi + \cancel{r^2} \sin^2\varphi}$$

$$= |r| \cdot \frac{\cos\varphi \cdot \sin^2\varphi}{\cos^2 + \sin^2} \leq$$

beschränkt

$\Rightarrow$ stetig

9b.)

$$f(x,y) = \begin{cases} \dfrac{xy^2}{x^2+y^4} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$$f\left(\frac{1}{n^2}, \frac{1}{n}\right) = \frac{\frac{1}{n^2} \cdot \frac{1}{n^2}}{\frac{1}{n^4} + \frac{1}{n^4}} = \frac{\cancel{\frac{1}{n^4}}}{\cancel{\frac{1}{n^4}} + \cancel{\frac{1}{n^4}}} = 1$$

$$f\left(0, \frac{1}{n}\right) = \frac{0 \cdot \frac{1}{n^2}}{0 + \frac{1}{n^4}} = 0 \quad \Rightarrow \text{unstetig}$$