Diskrete Tutorium 05062007

Note Title 05.06.2007

126.)

$$H(x) = \sum_{n=1}^{\infty} H_n \cdot x^n$$

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots \frac{1}{n} = \sum_{k=1}^{\infty} \frac{1}{k}$$

$$H(x) = \sum_{n=0}^{\infty} \left(\sum_{k=1}^{\infty} \frac{1}{k} \right) \cdot x^n = \frac{1}{1-x} \cdot \sum_{n=1}^{\infty} \frac{1}{n} \cdot x^n$$

$$\sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} a_n b_{n-k} \right) x^n = \left(\sum a_n x^n \right) \left(\sum b_n x^n \right)$$

$$a_k = \frac{1}{k} \qquad a_0 = 0$$

$$b_n = 1$$

$$\sum a_k x^k \sum b_n x^k$$

$$\frac{x^n}{n} = \int x^{n-1} \, dx$$

$$= \frac{1}{1-x} \cdot \int \sum_{n=0}^{\infty} x^n \, dx$$

$$= \frac{1}{1-x} \cdot \int \frac{1}{1-x} \, dx$$

$$= \frac{1}{1-x} \cdot \ln(1-x) \cdot (-1)^n + c \qquad / c = 0$$

127.) $A(z) = \sum_{n=0}^{\infty} k^2 x^n$

$$B(z) = A(z) \cdot \frac{1}{1-x}$$

$$z \frac{d}{dz}\left(z \frac{d}{dz} \sum_{n=0}^{\infty} z^k \right)$$

$$= \frac{z^2 + z}{(1-z)^3}$$

$$B(z) = \frac{z^2+z}{(1-z)^4} ; \quad \frac{A}{(1-z)^4} + \frac{B}{(1-z)^3} + \frac{C}{(1-z)^2} + \frac{D}{(1-z)}$$

$A = 2$
$B = -3$
$C = 1$
$D = 0$

$$\sum_{n=0}^{\infty} \binom{c+n-1}{n} a^n z^n = \frac{1}{(1-az)^c}$$

$$= 2 \cdot \sum_{n=0}^{\infty} \binom{4+n-1}{n} z^n - 3 \sum_{n=0}^{\infty} \binom{3+n-1}{n} z^n + \sum_{n=0}^{\infty} \binom{2+n-1}{n} z^n$$

$$= \sum_{n=0}^{\infty} \left(2 \cdot \frac{(n+3)(n+2)(n+1)}{3!} - 3 \frac{(n+2)(n+1)}{2!} + n+1 \right) z^n$$

$$= \frac{n(n+1)(2n+1)}{6} z^n$$

128.) $f(x,y)\cdot f(x^2,y)\cdot f(x^5,y)\cdot f(x^{10},y)$

$\Rightarrow$ 4 Möglichkeiten

130.) $(x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k}$

$$[x^n]\left(1+x^1+x^2+\dots+x^n\right)(1+x)^n$$

$$=[x^n]\left(1+x^1+x^2+\dots+x^n\right)\cdot\sum_{k=0}^{n}\binom{n}{k}x^k$$

$$=[x^n]\frac{1}{1-x}\cdot\sum_{k=0}^{n}\binom{n}{k}x^k$$

$$=\sum_{k=0}^{n}\binom{n}{k}=2^n$$