Diskrete Mathe Tutorium 13.03.2007

Bsp 8.)

$$
\begin{array}{ccc|ccc}
 & & & 0{,}5 & 0 & 0{,}5 \\
 & & & 0 & 0{,}5 & 0{,}5 \\
 & & & 0{,}25 & 0{,}25 & 0{,}5 \\
\hline
4000 & 4000 & 4000 & 3000 & 3000 & 6000
\end{array}
$$

$\Rightarrow$ bleibt const.

Bsp 9.)

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ 20 & 13 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 13 & 20 \\ 5 & 8 \end{pmatrix}$$

Bsp 10.) $A \cdot B \neq B \cdot A$

$$
\begin{array}{c|c}
 & \begin{pmatrix} e & f \\ g & h \end{pmatrix} \\
\hline
\begin{pmatrix} a & b \\ c & d \end{pmatrix} & \begin{matrix} ae+bg & af+bh \\ ce\ dg & cf+dh \end{matrix}
\end{array}
\qquad
\begin{array}{c|c}
 & \begin{pmatrix} a & b \\ c & d \end{pmatrix} \\
\hline
\begin{pmatrix} e & f \\ g & h \end{pmatrix} & \begin{matrix} ae+fc & be+fd \\ ga+hc & gb+hd \end{matrix}
\end{array}
$$

$$\begin{pmatrix} ae+bg & af+bh \\ ce\ dg & cf+dh \end{pmatrix} = \begin{pmatrix} ae+fc & be+fd \\ ga+hc & gb+hd \end{pmatrix}$$

$$\cancel{ae} + bg \overset{!}{=} \cancel{ae} + fc \Rightarrow c = b = 0$$

$$af + bh \overset{!}{=} be + fd \Rightarrow f(a-d) \overset{!}{=} b(e-h) \Rightarrow a \overset{!}{=} d$$

$$ce + dg \overset{!}{=} ga + hc \Rightarrow c(e-h) \overset{!}{=} g(a-d) \Rightarrow a \overset{!}{=} d$$

$$cf + \cancel{dh} \overset{!}{=} gb + \cancel{hd} \Rightarrow cf = gb \Rightarrow c = b = 0$$

$$\Rightarrow \forall B: A \cdot B = B \cdot A \; : A = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$$

$$\Leftarrow \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \cdot \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \quad ✓$$

11.)

$$\begin{array}{c|cc}
 & \cos\beta & \sin\beta \\
 & -\sin\beta & \cos\beta \\
\hline
\cos\alpha \quad \sin\alpha & a & b \\
-\sin\alpha \quad \cos\alpha & c & d
\end{array}$$

a: $\cos\alpha \cos\beta + (-\sin\alpha \sin\beta) = \cos(\alpha+\beta)$

b: $\cos\alpha \sin\beta + \sin\alpha \cos\beta = \sin(\alpha+\beta)$

c: $-\sin\alpha \cos\beta + (-\sin\beta \cos\alpha) = -\sin(\alpha+\beta)$

d: $-\sin\alpha \sin\beta + \cos\alpha \cos\beta = \cos(\alpha+\beta)$

$$A^{-1}: \frac{1}{\underbrace{\cos^2\alpha + \sin^2\alpha}_{=1}} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

12.) $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad B = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ → Aussehen der Inverse (Annahme)

$$\begin{array}{cc|cc}
 & & d & -b \\
 & & -c & a \\
\hline
a & b & ad-bc & -ad+ab \\
c & d & cd-cd & ad-bc
\end{array}$$

$$\Rightarrow ad - bc = 0 \qquad \Rightarrow A \cdot B = 0$$

$C \cdot A = I \qquad I \cdot B = B$

$C \cdot A \cdot B = B$

$C\,(A \cdot B) = B$

$C \cdot 0 = B$

$B = 0$ ↯ zu $B \neq 0$

13.) $A \cdot B = C \qquad B^{-1} \cdot A^{-1} = C^{-1}$

$C \cdot C^{-1} = I \qquad A \cdot I = I \cdot A = A$

$A \cdot B \cdot B^{-1} \cdot A^{-1} = I$

$A \cdot \underbrace{(B \cdot B^{-1})} \cdot A^{-1} = I$

$\underbrace{(A \cdot I\ A^{-1})} = I$

$I = I$ ✓

14.) aa.) $\det = -4$ über Sarrus

ab.) $\det = 0$ über Sarrus

$$\begin{vmatrix} 1 & 1 & 1 \\ -2 & -3 & 0 \\ 1 & 2 & -1 \end{vmatrix}$$

$\Sigma \quad 0 \quad 0 \quad 0 \Rightarrow$ linear abhängig $\Rightarrow \det = 0$