Diskrete Mathe

Notiztitel 04.03.2007

2a.) $\alpha \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

$$\begin{aligned} \alpha 1 + \beta 1 &= 1 \quad \Rightarrow \alpha = 1 - \beta \\ \alpha 1 - \beta 1 &= 5 \\ 1 - \beta - \beta &= 5 \\ -4 &= 2\beta \quad \Rightarrow \beta = -2 \end{aligned}$$

$$\alpha = 1 - (-2)$$
$$\alpha = 3$$

2b.) $\alpha \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$

$$\begin{aligned} \alpha + \beta &= -1 \quad \Rightarrow \alpha = -(1+\beta) \\ -1 - \beta - \beta &= 5 \quad \Rightarrow -2\beta = 6 \end{aligned}$$

$$\beta = -3$$

$$\alpha = -1 + 3$$
$$\alpha = 2$$

4a.)

$$\left| \begin{pmatrix} -1 & 3 & 2 \\ 2 & -1 & 2 \\ 3 & -2 & 1 \end{pmatrix} \right.$$

$$\left(\begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 2 \\ -1 & 2 & -2 \end{pmatrix} \right| \left. \begin{matrix} -1+(-2)+3 & 3+1-2 & 2-2-1 \\ 0 & 2 & 1 \\ -2+0+6 & 6+0-4 & 4+0+2 \\ 4 & 2 & 6 \\ 1+4-6 & -3-2+4 & -2+4-2 \\ -1 & -1 & 0 \end{matrix} \right|$$

4b.)

	1	2	-1
	-1	0	2
	1	1	-2
-1 2 3	-1-2+3 = 0	-2+0+3 = 1	1+4-6 = -1
3 -1 -2	3+1-2 = 2	6+0-2 = 4	-3-2+4 = -1
2 2 1	2-2+1 = 1	4+0+1 = 5	-2+4-2 = 0

5a.)

$$\begin{pmatrix} 1 & 3 & -2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 & 3 & 2 \\ 2 & 6 & -4 \\ 3 & 9 & -6 \end{pmatrix}$$

1×3 $\qquad$ 3×1 $\qquad$ 3×3

	-1
	2
	3
1 3 -2	-1+6-6 = -1

5b.)

$$\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \cdot (1, -2, 3) = \begin{pmatrix} 2 & -4 & 6 \\ -1 & 2 & -3 \\ 3 & -6 & 9 \end{pmatrix}$$

5c.)

	1
	-2
	3
-1 2 3	-1-4+9 = 4

5d.)

$$\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \cdot (2, -1, 3) = \begin{matrix} 2 & -1 & 3 \\ -4 & 2 & -6 \\ 6 & -3 & 9 \end{matrix}$$

6a.)

1	4	3	1	4+4 = 8	3+8+3 = 14
0	1	2	0	1	4
0	0	1	0	0	1

6b.)

1	0	0	1	4	3
4	1	0	4	16+1 = 17	12+2 = 14
3	2	1	3	12+2 = 14	9+4+1 = 14

6c.)

1	2	3	1	2+2 = 4	3+8+3 = 14
0	1	4	0	1	4+4 = 8
0	0	1	0	0	1

6d.)

1	0	0	1	2	3
2	1	0	2	4+1 = 5	6+4 = 10
3	4	1	3	6+4 = 10	9+16+1 = 26

7a.) $\begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}^{25}$

	1	0	
	x	1	
1	0	1+0 = 1	0+0 = 0
x	1	x+x = 2x	0+1 = 1

...

	1	0
	3x	1

$\Rightarrow \begin{pmatrix} 1 & 0 \\ x \cdot 25 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}^{-25} = \begin{pmatrix} 1 & 0 \\ -x & 1 \end{pmatrix}^{25} = \begin{pmatrix} 1 & 0 \\ -25 & 1 \end{pmatrix}$