Diskrete Mathe KV 25.06.2007

Note Title 25.06.2007

$$\frac{1}{z-z_0} = -\frac{1}{z_0} \cdot \sum_{k=0}^{\infty} \left(\frac{z}{z_0}\right)^k$$

$$s_n = \sum_{k=1}^{n} k = 1+2+\ldots+n \quad , s_0 = 0$$

$$S(z) = \sum_{l=0}^{\infty} s_k s^k = \frac{1}{1-z} \cdot \sum_{l=0}^{\infty} k z^k$$

$=$

$$b_n = \sum_{k=0}^{n} a_n$$

$$A(z) = \sum_{k=0}^{n} a_k z^k$$

$$B(z) = ?$$

allgemein: $\left(\sum_{k=0}^{\infty} a_k z^k\right)\left(\sum_{l=0}^{\infty} c_k z^k\right) = \underbrace{\sum_{n=0}^{\infty} \sum_{l=0}^{n} (a_k c_{n-l}) z^k}_{\text{Cauchy Produkt}}$

über Cauchy $B(z) \Rightarrow b_n = \sum c_{n-k} \cdot z^k \Rightarrow c_n = 1$

$$\sum_{l=0}^{\infty} c_k z^k = \sum_{k=0}^{\infty} 1 \cdot z^k = \frac{1}{1-z} \Rightarrow B(z) = \frac{1}{1-z} \cdot A(z)$$

$$\frac{1}{1-z} \cdot \sum_{k=0}^{\infty} k z^k \qquad \Big| \frac{d}{dz} \sum z^k$$

$$\frac{1}{1-z} \cdot \frac{d}{dz}\left(\frac{1}{1-z}\right) \cdot z \qquad \Rightarrow \sum k z^{k-1} = \underbrace{\frac{d}{dz}\left(\frac{1}{1-z}\right)}_{\frac{1}{(1-z)^2}}$$

$$= \frac{1}{1-z} \cdot z \cdot \frac{1}{(1-z)^2} = \frac{z}{(1-z)^3}$$

$$= \frac{A_1}{(1-z)} + \frac{A_2}{(1-z)^2} + \frac{A_3}{(1-z)^3}$$

$$z = A_1(1-z)^2 + A_2(1-z) + A_3(1-z)$$

$$z = 1 \Rightarrow A_3 = 1$$

$$\frac{d}{dz} : 1 = 2(1-z)A_1 - A_2$$

$$z = 1 \Rightarrow A_2 = -1$$

$$\frac{d}{dz} = 0 = -2A_1 \Rightarrow A_1 = 0$$

$$S(z) = \frac{1}{(1-z)^3} - \frac{1}{(1-z)^2}$$

$$= \frac{1}{2} \sum_{k=0}^{\infty} (k+1)(k+2) z^k - \sum_{k=0}^{\infty} (k+1) z^k$$

$$= \sum_{k=0}^{\infty} \left[\frac{1}{2} (k+2) - 1 \right] (k+1) \; z^k$$

$$= \sum_{k=0}^{\infty} \frac{1}{2} k \, (k+1) \; z^k$$

$$\left.\begin{aligned} (1+z)^{\alpha} &= \sum_{k=0}^{\infty} \binom{\alpha}{k} z^k \\ \frac{d^2}{dz^2}\left(\frac{1}{1-z}\right) &= \frac{2}{(1-z)^3} \end{aligned}\right\} \text{allgemein}$$

alternative:

$$s_n = 1+2+ \dots + n = s_{n-1} + n \quad , \; s_0 = 0$$

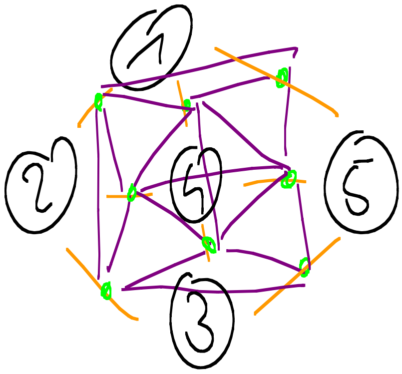
$$S(z) = \sum_{n=1}^{\infty} s_n z^n = \sum_{n=1}^{\infty} (s_{n-1} + n) \, z^n$$

$$= \underbrace{\sum_{n \geq 1}^{\infty} s_{n-1} \; z^n}_{z \cdot \sum_{n=0}^{\infty} s_n z^n} + \underbrace{\sum_{n=1}^{\infty} n \, z^n}_{\frac{z}{(1-z)^n} - 0 \cdot z^0}$$

$$= z \cdot S(z) + \frac{z}{(1-z)^2}$$

$$S(z)\,(1-z) = \frac{z}{(1-z)^2}$$

$$S(z) = \frac{z}{(1-z)^3}$$ weiter wie vorher

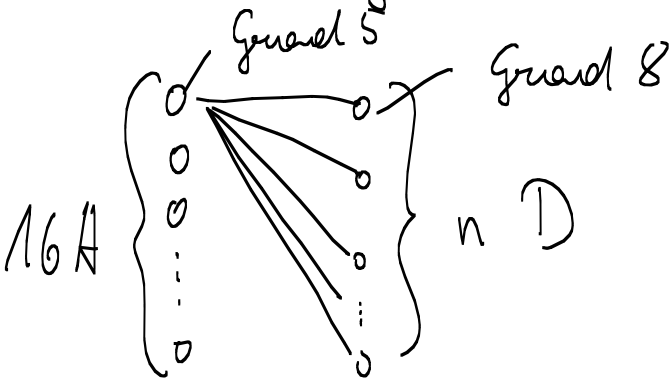


$V = (\{1, 2, 3, 4, 5\})$

$E = (\{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}, \{1,4\}, \{4,3\}, \{2,4\}, \{4,5\})$

Kantengraph: E wird zu V

Z12.)



$16 \cdot 5 \overset{!}{=} 8 \cdot n \Rightarrow n = 10$

Bipartiter Graph: $V = V_1 \dot{\cup} V_2$

Es gibt nur E von V_1 nach V_2, sonst nichts

$\underbrace{|E|}_{\text{Anzahl Kanten}} = \sum_{v \in V_1} \underbrace{d(v)}_{\text{Grad der Knoten}}$

$$\sum_{v \in D} d(v_1) = \sum_{v \in H} (d\, v_2)$$

$$8 \cdot |D| = 5 \cdot |H| \Rightarrow |D| = \frac{5}{8} \cdot |H|$$

$$= \frac{5}{8} \cdot 16$$

$$= 10$$